PRESCHOOL MATH CONVERSATIONS: AN ETHNOGRAPHIC STUDY OF ONE
CLASSROOM

by

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DEDICATION

For my father, who does not think himself a scholar, but who has the love, energy, and grit to make up for this supposed lack. You offered me every kind of support you could at any hour of any day. I am so glad that this dissertation afforded us the opportunity to think, talk, and laugh together. I could not have done it without you.
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ABSTRACT

BETSY LAUREN GANSBORG
PRESCHOOL MATH CONVERSATIONS: AN ETHNOGRAPHIC STUDY OF ONE CLASSROOM
Under the direction of WILLIAM O. LACEFIELD, III, Ed.D.

U.S. students fall behind those in many other developed countries in international benchmarking exams for mathematics. Math instruction in the U.S. generally begins in earnest in first grade even though current research in neuroscience and cognitive science shows that preschool-age children are ready to learn about mathematics. Education research links the importance of verbal communication and mathematics for children in K-12 settings. However, little research exists on verbal communication in mathematics lessons at the preschool level.

The current ethnographic study explored mathematical conversations in one private preschool classroom. Participants were a Taiwanese-American teacher and her 10 high-SES, ethnically and linguistically diverse students. Data included observations of math lessons that took place in the classroom and interviews with the teacher. Data were analyzed using the constant comparison method.

The results showed that the teacher progressed through three phases as she developed a distinct conversation style that she used with her students. Ten teacher conversation strategies were identified. Employment of these conversation strategies prompted teacher and students to co-construct sociomathematical norms. As a result of
this co-construction, conversation increasingly became developmentally appropriate.

Implications for instruction include the imperative that teachers of preschool-age children engage these students in productive mathematical conversation using research-based conversation strategies. Recommendations for future research include the quantitative measurement of how teacher conversation strategies impact student test scores.
CHAPTER 1
INTRODUCTION TO THE STUDY

Evidence from the fields of cognitive science (Barth, La Mont, Lipton, & Spelke, 2005; Clements & Sarama, 2007; Huttenlocher, Jordan, & Levine, 1994; Klein & Bisanz, 2000; Lee & Ginsburg, 2007b; Mix, 1999, 2002; Mix, Huttenlocher, & Levine, 1996) and neuroscience (Cantlon, Brannon, Carter, & Pelphrey, 2006; Dehaene, Piazza, Pinel, & Cohen, 2003; Rivera, Reiss, Eckert, & Menon, 2005) have shown clearly that preschool-age children are capable of reasoning about mathematical concepts. They possess the cognitive equipment to reason numerically (Dehaene & Cohen, 1995; Dehaene et al., 2003; Dehaene, 2011).

Furthermore, studies of children slightly older than preschoolers (kindergarteners, first graders, and second graders) demonstrate that high-quality conversations enable young children to develop numerical reasoning. Participants were able to solve difficult problems through talk (Cobb & Bowers, 1999; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Cobb, Stephan, McClain, & Gravemeijer, 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996). The research by Cobb and colleagues served as the conceptual framework for the current study on mathematical conversations in a preschool classroom. Cobb’s research offered qualitative portraits of high-quality mathematics conversations with young children (K-2nd). In my thorough searches, I found no similar research on mathematical conversations for preschool-age students. In this study, I
extended the research on high-quality mathematical conversations and resulting mathematical reasoning abilities for preschool-age children.

Mathematics and U.S. Competition on the World Stage

Recent research shows that teachers and caregivers of preschool-age children doubt the appropriateness of mathematical instruction for children of this age (Lee & Ginsburg, 2007a). In my personal experience, I can say that many of my peers in the early childhood education community also dislike teaching math or feel they are ill-equipped to design this type of instruction. I propose that this fear of math may prove devastating for our society and economy. In the twentieth century, it may have been acceptable for the math-phobic educational community (Lee & Ginsburg, 2007a) to ignore the mathematical needs of students. However, in the 21st century, the United States must overhaul teaching practices to keep up with the rest of the world. Students from the United States routinely perform far below students from Finland and East Asian countries in math (Fleischman, Hopstock, Pelczar, & Shelley, 2010). Research has demonstrated that change begins in early childhood (Barnett, 2007). Without a strong foundation of mathematical knowledge and reasoning skills, teachers of older children have nothing to build upon.

On international benchmarking tests, U.S. students score far lower than many of their foreign counterparts in reading literacy, science literacy, and especially in mathematical literacy. As indicated in the latest Program for International Student Assessment (PISA) results, U.S. fifteen-year-olds ranked 40th out of the 68 other Organization for Economic Cooperation and Development (OECD) nations in
This performance gap is alarming to lawmakers, business leaders, and educationists alike. These interested parties are concerned that consistent low math scores might impair citizens’ ability to innovate in today’s technological society (Darling-Hammond, 2010). Thus, poor mathematical literacy threatens the United States’ competition on the world stage.

Evidence from Neuroscience, Cognitive Science, and Education

However, research suggests reasons to be optimistic. The gap is small between U.S. children and their counterparts in East Asian countries when these children are four years old (Ginsburg, Choi, Lopez, Netley, & Chi, 1997; Huntsinger, Jose, Liaw, & Ching, 1997). However, it grows by kindergarten (Stevenson et al., 1990) and by first grade (Zhou, Peverly, & Lin, 2005). These findings suggest that in countries where children perform well, this performance is possibly due to successful early childhood interventions. It is possible for the United States to take inspiration from this success and design culturally-appropriate, successful early childhood interventions in the United States as well. Research is needed to understand what effective mathematics instruction looks like at the preschool level. This study was designed to make this inquiry.

In the increasingly technological world, American students risk falling behind in the race to innovate. Researchers and educationists have no choice but to teach science, technology, engineering, and mathematics together. In an increasingly technologically sophisticated society, these subjects can no longer be separated. They are pieces of one language of innovation. All students must enter today’s workforce fluent in this language
(Darling-Hammond, 2010) in order to collaborate or compete. Mathematics is both a civil right (Kress, 2005) and key to U.S. success on the world stage.

Research has demonstrated that early investment in children’s education improves a nation’s economy over time (Barnett, 2007). This research has demonstrated that for every dollar invested in early childhood education, four to eight dollars is spared in expenditures in future social programs. Effective early childhood education leads also to higher employment rates (Gilliam & Zigler, 2000). It appears that effective early childhood interventions may be a silver bullet for improving the economy.

Concerns about the viability of the United States in global markets come simultaneously with cognitive science findings that preschool-age children already possess great potential to learn complex mathematics (Barth et al., 2005; Butterworth, 2005; Dehaene, 2011; Dehaene et al., 2003). These conclusions have been supported by findings in neuroscience that children possess the anatomical equipment to develop number sense early in life. Although still in its infancy, this neuroscience research has overturned much of that which was traditionally believed about early childhood abilities to learn mathematics. This research provides inspiration for educationists to approach mathematics instruction in new and innovative ways. Specifically, it encourages policy makers, teachers, and curriculum designers to build high-quality conversation into mathematics instruction.

When children learn math early, they continue to make mathematical progress for the years to come (Arnold, Fisher, Doctoroff, & Dobbs, 2002; Duncan et al., 2007). Furthermore, researchers have found that to neglect the mathematics education of
preschool-age children can result tragically in their later school failure (Lee & Burkam, 2002). Opportunities are lost, and children may fall further and further behind (Starkey & Klein, 2000). These are both personal losses as well as losses to our national and global economy.

Neuroscience evidence reveals that preschool-age children—as well as babies (Geraci & Surian, 2011; Starr, Libertus, & Brannon, 2013) and animals (Beran, 2004, 2007)—are cognitively equipped to grasp numbers and operations that are more sophisticated than previously assumed (Dehaene, 2011; Klein & Bisanz, 2000). Neuroscientists Cantlon and colleagues (2006) found that the same area of the brain activated in adult brains during mathematical problem solving is also activated in the brains of four year olds. It appears clear that the neurological basis for comprehension of number is developed even before formal schooling begins.

Furthermore, research in psychology (Vygotsky, 1934/1978) and linguistics (Gee, 2014a) has revealed that conversation is linked to learning and concept formation. Education research has also shown that conversation is linked to mathematical competence (Lampert, 2001; McClain & Cobb, 2001; O’Connor 1998). The picture that emerges from research recommendations is the need for discourse-rich preschool classrooms. Preschool teachers should thus be equipped with pedagogical tools to promote mathematical conversations with children at the top of their abilities. These teachers should be aware of children’s mathematical strengths and weaknesses so that they can plan for conversations that optimally capitalize on their abilities. My experience has shown me that conversation is a must for children to learn
mathematics, although it may not seem obvious. Research and my own experience has proven that language is important for learning math.

Animals, infants, and young children can all perceive quantity in a rough, intuitive capacity long before they begin assigning number to quantities (Dehaene, 2011). However, the understanding that 3 carrots, 3 bell chimes, and 3 puppet jumps all have something in common (the number 3)—is a matter of abstraction. Vygotsky (1934/1978) wrote, “The internalization of socially rooted and historically developed activities is the distinguishing feature of human psychology, the basis of the qualitative leap from animal to human psychology” (p. 57). Language permits higher-order thinking and thus more formal mathematical understandings of everyday experiences.

Conversation about numbers and operations are necessary for young children to make the leap from intuition into abstraction. The number 3 is, in fact, an abstract category that unifies all of these sets of disparate items. As Vygotsky (1934/1986) wrote in *Thought and Language*, “A word does not refer to a single object, but to a group or to a class of objects” (p. 6). Talk is not the only ingredient necessary for abstraction to happen. Symbolic written notation, the drawing of pictures, and physical activities all play a role in the mathematization of intuitive experience (Walkerdine, 1991). However, conversation is vital for the development of mathematical competence (Walshaw & Anthony, 2008) because children internalize this talk so that it becomes their inner reasoning (Vygotsky, 1934/1978, 1934/1986).
Teachers and Caregivers’ Attitudes about Early Childhood Math

It seems that in spite of research encouraging teachers to include mathematics in their early childhood programs, teachers do not talk about mathematics when asked what is important in early childhood curricula (McMullen et al., 2005; Stipek & Byler, 1997). Without teachers’ support of implementation of research recommendations, the United States cannot reach its economic goals (Darling-Hammond, 2010). Teachers may have legitimate reasons to feel confused about how to fit math into the curriculum.

Although National Council of Teachers of Mathematics (NCTM) (2000) published its landmark document, *Principles and Standards for School Mathematics*, on principles and standards to include prekindergarten, these standards provide only a rough outline. *Principles and Standards for School Mathematics* (NCTM, 2000) does not dictate lessons to teach in order to reach the standards.

In 2007, the National Research Council (NRC) created the Committee on Early Childhood Mathematics. In this synthesis of literature (NRC, 2009), the authors focused on two areas of mathematics as most important for preschool-age children: number sense and geometry. Like NCTM (2000), NRC emphasized verbal communication as vital for development of mathematical proficiency. With these and other documents like them, teachers possess rough guidelines for how to prepare preschool children mathematically, but no immediate instructions for how to implement standards. The time is appropriate for research on specific mathematical teaching practices that will benefit young children. This study addressed this need.
Research shows that teachers and parents alike often doubt the appropriateness of mathematics instruction in early childhood classrooms (Blevins-Knabe, 2016; Cannon & Ginsburg, 2008; Lee & Ginsburg, 2007a). Cannon and Ginsburg (2008) found that more than half of a diverse group of 37 mothers believed it was more appropriate to teach language to, and to support social and emotional development of, their preschool-age children than it was to teach them mathematics. Lee and Ginsburg (2007a) interviewed 30 prekindergarten teachers on their beliefs about teaching mathematics to their students. The 15 teachers of publicly funded prekindergarten believed mathematics should be planned and taught explicitly in order to prepare children for kindergarten. The 15 teachers of privately funded prekindergarten believed mathematics experiences should be initiated by the child and not explicitly planned or executed. This research illustrates the confusion and differences of opinion surrounding early childhood mathematics. Many teachers feel uncomfortable teaching math (Stipek & Byler, 1997). Some teachers believe it to be inappropriate to teach to young children mathematics. It is encouraging that some teachers believe mathematics should be explicitly taught to young children. This study offered more evidence for teachers to use when making their judgments about the legitimacy of teaching young children math.

This study will help resolve some teachers’ mixed emotions when it comes to teaching early childhood mathematics. I purposely selected a preschool teacher who believed that mathematics should be taught in a deliberate, organized manner to preschoolers. She also believed that conversation is an important part of teaching
mathematics. The qualitative portrait I painted of this teacher and her students will help teachers and other caregivers to conceive of mathematical instruction for preschoolers.

The Need for Qualitative Research

Qualitative research on math conversations in a preschool classroom is necessary. Before anything is known on a subject, quantitative research is not much help. It is not possible to measure relationships between variables when the variables have yet to be delineated. Open-ended qualitative research is necessary to open lines of inquiry for quantitative researchers to follow. Open-ended qualitative research is also necessary so that the public can begin to imagine what these conversations sound like. From my work with preschool-age children, I can say that mathematical conversations with this age group are possible, legitimate, and urgently necessary.

It is important to remember that, even with the goal of increasing sophistication of early childhood mathematics instruction, instruction must remain developmentally appropriate (National Association for the Education of Young Children [NAEYC], 2009). Preschoolers learn differently than do kindergarteners and older children. It is important for them to develop solid relationships with each other and with their caregivers and also to experience autonomy so they can develop intellectual independence. They must be free to explore manipulatives and to move around the room rather than sit still in chairs filling out worksheets. In addition, they rely heavily not just on verbal cues but also on nonverbal cues such as gestures and facial expressions (Birch, Akmal, & Frampton, 2010). They depend on adults to model actions as well aid in their understanding (Wohlwend, 2009). The thick, rich descriptions (Geertz, 1973) that
qualitative research yields showed the nuances of how one teacher orchestrated premath and mathematical conversations in her preschool classroom.

It was not currently known whether preschool-age children would benefit from mathematical conversations in the way that K-12 children do (Walshaw & Anthony, 2008). If mathematical conversations are appropriate for preschoolers, it was not known of what these conversations should consist. These pressing questions created a new qualitative study on the subject of early childhood mathematical conversations a necessity. Our ignorance on the topic of early childhood mathematics could amount to a momentous lost opportunity. U.S. mathematics education is in a state of emergency. The United States is today losing its competitive edge on the world stage because of its citizens’ mathematical illiteracy (Darling-Hammond, 2010). Perhaps sophisticated mathematical ideas can be taught earlier. Research is necessary to discover how mathematics should be taught to the youngest children. If it is possible to teach mathematics earlier, it is an opportunity that the United States cannot afford to miss.

Overview of Theoretical and Conceptual Frameworks of the Study

Teachers and students co-construct mathematical concepts through conversation (Yackel & Cobb, 1996). They do so as members of a classroom microculture. The classroom microculture forms the immediate conceptual framework of the current study. As teachers and students converse, they co-construct sociomathematical norms, or norms for thinking and speaking mathematically in a social setting (Lopez & Allal, 2007; Yackel & Cobb, 1996). This activity of co-construction can be understood within the larger theoretical concept that individuals participate peripherally in a community of
practice as they are apprenticed by more experienced members of that culture (Lave & Wenger, 1991). This post Vygotskian theory is called situated learning. In the case of the classroom I studied, both the teacher as well as the students who were more experienced in mathematical reasoning and speech acted in the role of mentors to those students less experienced with mathematics. These less experienced students act in the role of apprentices.

Ultimately, all children are apprenticed by the teacher. However, it should be emphasized that some children enter preschool with more experience than others in the use of academic language. Research reveals that some children enjoy more experience with academic language at home than do less advantaged peers (van Kleeck, 2014). Thus, a child may act as a more experienced other over the course of the co-construction of sociomathematical norms (Lopez & Allal, 2007; Yackel & Cobb, 1996).

Situated learning is a post Vygotskian theory. It assumes many pieces from Vygotskian theory, including the idea that individuals’ mathematical cognition is shaped by social and especially linguistic exchanges (Vygotsky, 1934/1978, 1934/1986). As children gain fluency in mathematical language, they gain more central status in the community of practice (Lave & Wenger, 1991). Basically, this is how the work of Vygotsky; Lave; Wenger; Cobb and colleagues; and van Kleeck and colleagues fit together to form the theoretical and conceptual frameworks for this study. Two other pieces that cut across this nested system are children’s linguistic differences (Lagrander & Reid, 2000) and teacher conversation strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012; Wood, 1989).

Sociomathematical norms do shape individual children’s reasoning abilities, but what happens when children’s linguistic differences limit their access to classroom conversations? This difference may prohibit them from developing the same norms and thus, the same reasoning as other students in the class (Lagrande & Reid, 2000). Exclusion from initial authorship of the classroom norms can lead to a vicious cycle of continued exclusion. However, teacher conversation strategies can offer a solution in that they help extend conversation to include more voices (Chapin & O’Connor, 2007; Temple & Doerr, 2012; Wood, 1989).

The Situated Learning Lens

The community of practice model (Lave & Wenger, 1991) becomes very useful in light of research on linguistic differences in the classroom (van Kleeck, 2014). Some students who are fluent in school-appropriate language immediately access conversations with teachers. Those less fluent in school-appropriate language are excluded from conversation. Differential home experiences can lead to this discrepancy (van Kleeck, 2006, 2008, 2014). Some parents socialize their children into ways of speaking that resemble academic language before formal schooling begins. Those students who are not fluent in academic language may remain on the margins of the classroom community of practice while those already fluent in academic language move progressively from the periphery into central participation in the classroom community.
In this linguistic view of the classroom, the zone of proximal development (ZPD) (Vygotsky, 1934/1978) can be located in those moments where students encounter a way of speaking that is foreign to them. Teacher conversation strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012; Wood, 1989) can help students bridge these gaps. However, without deliberate planning on the teacher’s part, these students may remain on the periphery. Those already fluent in school-appropriate language can be pushed into more formal ways of thinking and talking about mathematics through high-quality experiences. Those less fluent require scaffolding (Wood, Bruner, & Ross, 1976), or else new forms of language will remain outside of their ZPD. New language must be offered within their ZPD so that they may make this language their own. With proper scaffolding, these students may even join in co-constructioning classroom norms for reasoning mathematically. Thus, the classroom microculture may reflect these students’ own home cultures so that they will continue to succeed within this context. It is up to teachers to build bridges so that children can translate meanings from their home languages and everyday ways of speaking to formal mathematical ways of speaking. It can make the difference between the student moving towards central participation or remaining on the periphery.

Students enter the classroom with different linguistic needs. Linguistic minority students require different instruction than linguistic majority students in order to be academically successful (Heller, 2006; van Kleeck, 2014). Those students who need the most practice speaking school-appropriate language often receive the least. Those students already advantaged by home experience are often favored by the teacher.
linguistically. They receive more practice in academic language. As will be explained in
the subsequent section on classroom microcultures, this linguistic practice leads to the
construction of a classroom culture that advantages certain students more and more as
time passes.

What role do teachers play? Teachers can be considered the central participants
in the classroom community of practice (Walshaw & Anthony, 2008; Yackel & Cobb,
1996). All of the students enter the classroom as peripheral participants in the emerging
community of practice. However, because of the sometimes prejudicial teacher behavior
described previously, privileged students may move more quickly to the position of
central participants. The less advantaged students remain on the periphery. Thus, the
reproduction of the group culture will reflect the beliefs and preferences of the
advantaged children. The ultimate result of this work will be a culture that further
excludes those children who were disadvantaged in the beginning. This particular
conclusion is not taken from any specific study. It is rather my novel synthesis of the
literature on situated learning and on linguistic difference that I have reviewed thus
far. The culture of a classroom creates the context in which students absorb ways of
walking, talking, and practicing (Lave & Wenger, 1991). Without the access to the
cultural-specific ways of being, doing, and speaking, situated learning is not possible.

In this study, I examined ways in which classroom culture was constituted
through group conversation and then how this group conversation shaped individual
cognition. I also qualitatively examined the differential effects group conversation had
on linguistic majority versus linguistic minority students. There are practices that can
disrupt this cycle of inequality (Chapin & O’Connor, 2004; Temple & Doerr, 2012). Research described some teacher conversation strategies that can be employed to extend mathematical conversations, include more students, make it clear that all of these voices are valued, and explicitly highlight expectations for formal mathematical communication. I will outline different models for teacher conversation strategies in Chapter 2.

Conceptual Underpinnings: The Work of Cobb and Colleagues

Situated learning serves as a theoretical framework within which to understand the concept of classroom microcultures. The construct of classroom microcultures served as the immediate conceptual framework for the current study. Paul Cobb and colleagues (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; McClain & Cobb, 2001; Yackel & Cobb, 1996) first developed the concept of classroom microcultures. Cobb and colleagues’ research served as a useful conceptual framework for academic language (Straehler-Pohl, Fernandez, Gellert, & Figueiras, 2013; van Kleeck, 2014), and teacher conversation strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012). These interrelated areas of research will be synthesized in detail in the following chapter.

Beginning in the 1980s, Cobb and colleagues began studying and writing about early elementary mathematics classrooms (Cobb & Steffe, 1983; Cobb, Wood, Yackel, Wheatley, & Merkel, 1988). Starting in the 1990s, these researchers (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996) introduced the terms classroom microculture and sociomathematical norms. The
term *sociomathematical norms* refers to the social norms specifically related to ways members of the group reason mathematically together, especially in group conversation. These norms are co-constructed by the members of the classroom microculture.

Cobb and colleagues adapted Lave and Wenger’s (1991) concept of the community of practice and adapted it for the purposes of analyzing mathematics classrooms. Borrowing on the idea of the community of practice, Cobb and colleagues called the mathematics classroom a classroom microculture. This nomenclature emphasized the small-scale of the mathematics classroom, as well as its nature as a culture unto itself complete with norms. However, they distinguished between social norms and sociomathematical norms. Certain social norms governed all academic classrooms, such as taking turns talking and offering evidence for one’s opinions.

Sociomathematical norms were of particular interest for the current study. As teachers and students converse about mathematical problems, they co-construct norms specifically for reasoning mathematically in their particular context. The mathematics classroom is situated within the larger culture of mathematical activity, which includes innovators and professional mathematicians. The mathematics classroom is both a microcosm of this activity, as well as a unique manifestation of that larger culture. The norms devised in that context reflect larger math culture, but also bear a unique character because they will have been constructed by the unique members of the classroom. It is important to note that Cobb and colleagues defined sociomathematical norms very narrowly, defining them as the norms used exclusively in mathematical activity. Later
researchers broadened the definition to include norms that may be used in other academic areas as well (Lopez & Allal, 2007). I made use of this latter, broader definition of sociomathematical norms.

As the group makes use of these sociomathematical norms, individuals internalize them such that they also shape the individual student’s independent work. Vygotsky (1934/1978) explained, “Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological)” (p. 57). Therefore, Yackel and Cobb (1996) suggested that once students engaged in sociomathematical norms, they internalized these norms, thus, the norms became the students’ own cognitive norms for reasoning mathematically. The key is for students to engage in this process. Some students engage in more math talk in their homes and thus come to school ready to engage in math talk. Other students have less experience with this language at home, and thus come to school less able to engage (Lagrandre & Reid, 2000; van Kleeck, 2014). It is up to the teacher to use conversation strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012; Wood, 1989) to bring all the students into the conversation so that all students will develop mathematical language and competence.

Findings on sociomathematical norms have important implications for designing appropriate curriculum for linguistic minority students, although research has not made this connection explicit. I have made what I believe is a novel connection between the literature on sociomathematical norms (Yackel & Cobb, 1996) and the literature on linguistic minority students (Lagrandre & Reid, 2000). Teachers engage in less
conversation with linguistic minority students (Barletta, 2008), and thus linguistic minority students lose in two ways. First, they receive the least amount of practice in academic conversation even though they are the ones who need it the most. Second, because their voices are not heard, they do not participate as actively in co-constructioning the sociomathematical norms for their classroom. Linguistic majority students enjoy the most air time, and thus participate most actively in co-constructioning sociomathematical norms. As the days of the school year go by, linguistic minority students find themselves in a culture in which norms for reasoning have been authored by their more privileged classmates. Because linguistic minority students were not the authors of these norms, they may find themselves increasingly alienated from the group’s ideas about correct reasoning.

Summary of Theoretical and Conceptual Frameworks

The technique of teacher talk moves (Chapin & O’Connor, 2004) offers hope that this inequitable cycle can be disrupted. Teachers can use a number of conversation strategies to extend the conversation and ensure that more students contribute. Engaging linguistic minority students (O’Neal & Ringler, 2010) in academic conversation ensures that they will have an opportunity to translate everyday ways of reasoning into more formal methods of reasoning (Cobb et al., 2001; Yackel & Cobb, 1996). These students will thus be active in co-constructioning sociomathematical norms. They will also become capable of developing academic language (van Kleeck, 2014). In the current study, I investigated the conversation strategies (Chapin & O’Connor, 2007) that a preschool teacher used to assist students to develop these competencies within their ZPD
I also investigated the subsequent mathematical development of the teacher’s ten students. As the teacher socializes students into appropriate mathematical beliefs, practices, and talk, these students are able move into more central participation in the mathematics community (Lave & Wenger, 1991). Central participation results in academic achievement for these students who would have otherwise remained on the periphery. As I have outlined in the theoretical and conceptual framework, teachers and students co-construct ways of thinking and speaking mathematically as they reproduce mathematical culture at the classroom level (Cobb et al., 2001; Yackel & Cobb, 1996). Broader mathematical culture includes the work of professional mathematicians, the many innovators who use mathematics in the workplace, and classrooms in which mathematics is taught. Teachers and students bring novel interpretations to the practice of mathematics. And most importantly for this study, they negotiate unique ways of speaking and reasoning about mathematics. This study captured some of the unique ways the participants interpreted mathematical language and reasoning.

Statement of the Problem

Evidence from the field of neuroscience has revealed that babies and young children possess the neurological basis to learn mathematics (Cantlon et al., 2006). Prominent neuroscientist, Stanislas Dehaene (2011) identified the anatomical substrate in humans that is associated with the ability to grasp and manipulate number. Children are capable of learning about number sense and operations at an early age (Barth et al., 2005; Cantlon et al., 2006; Huttenlocher et al., 1994; Klein & Bisanz,
Students in Finland and East Asian countries are routinely outperforming students from the United States in mathematics (Fleischman et al., 2010). In today’s technological society, U.S. citizens must be math literate in order to succeed and innovate. The United States may fail to compete with these other countries on the world stage because of the failure of schools to educate students in mathematics (Darling-Hammond, 2010). It is urgent that researchers produce answers to these problems.

Despite research demonstrating children possess the cognitive equipment to represent and manipulate number (Cantlon et al., 2006) and research capturing young children solving numerical problems (Barth et al., 2005; Huttenlocher et al., 1994; Klein & Bisanz, 2000; Mix, 1999, 2002; Mix et al., 1996), some early childhood caregivers are math phobic (Blevins-Knabe, 2016; Cannon & Ginsburg, 2008; Lee & Ginsburg, 2007a). In my experience, mathematics instruction may have been lackluster for the previous generation of children who are now adult teachers. Thus, young children may inherit this legacy of math phobia unless researchers can suggest strategies and tools that teachers can feel confident in using. This generation of teachers cannot afford to let math phobia prevent them from teaching young children mathematics.

Extensive research exists on the importance of conversation for the development of mathematical knowledge and competence (Walshaw & Anthony, 2008). Discourse-rich mathematics classrooms produce mathematically literate students. Useful research exists on the teacher conversation strategies teachers can use with confidence to enhance communication experiences during math lessons. However, research is lacking on
teacher conversation strategies used in preschool classrooms. Although some rich research exists on math talk in early childhood classrooms (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; McClain & Cobb, 2001; Yackel & Cobb, 1996), none of the studies pertained to preschool-age children.

The research conducted by Cobb and colleagues suggests that successful mathematical conversations focus explicitly on how children should engage in group conversation about numerical problems. Findings show that children internalize these conversations. Group conversation regulates individual reasoning, improving it and assisting students in gaining mathematical proficiency. It is urgent that researchers explore preschool classrooms in order to understand the ways in which teachers and students successfully co-construct ways of thinking and talking about mathematical topics. This research will help preschool teachers begin to imagine ways they can initiate similar conversations in their classrooms. This study produced suggestions for specific strategies that teachers can use. This qualitative study proved a strong step in the right direction for understanding preschool math conversations.

Early childhood mathematics instruction is critical (NAEYC, 2010; NCTM, 2001, 2003). However, parents and teachers alike are ignorant of what mathematics instruction should look like for young children. Our ignorance on this topic may cause U.S. citizens to lose economic opportunities in the world economy (Darling-Hammond, 2010). In order to innovate in a technological world, students of the 21st century must enjoy successful first experiences with mathematics. Teachers and researchers must understand what such successful experience should look like so that they can provide it.
Purpose of the Study

In order to provide mathematics instruction that is appropriate for the development of preschoolers, strong preliminary qualitative research is required in this area. This study specifically examined one key aspect of mathematics classroom practice: conversation. This study examined a complex social context in which conversations unfolded. Methodologically, I was intent upon balancing the tension between the open-endedness that qualitative research requires and the theoretical and conceptual framework I brought to bear upon the study. I employed open-ended qualitative methods (observations and interviews) to gather preliminary data on a topic researchers know so little about: mathematical conversations in a preschool classroom.

Although I used methods designed to be open-ended, I was cognizant of data that fit or did not fit with constructs from previous literature on mathematical conversations at the K-2nd grade level. Specifically, I investigated the sociomathematical norms constituted by initial conversation. I next examined how they governed the conversations that occurred subsequently. I paid special attention to the strategies that the teacher used to include all children in the conversation, as well as the challenges the teacher faced. Finally, I examined the manner in which group conversation shaped individuals’ subsequent reasoning about premath and mathematical concepts.

This study is important for both teachers and researchers. For teachers who regard mathematics with fear or confusion, this study will offer a portrait of one teacher’s strategic mathematics instruction. This portrait will help other teachers set clear goals for their own instruction, especially in planning for productive conversations. For
researchers, this study will offer preliminary qualitative recommendations to open up other lines of inquiry. These lines of inquiry can be pursued by quantitative researchers investigating the relationships between relevant variables. This study may be helpful in delineating the variables for further study. It will thus begin to fill the void in the research on mathematical conversations in the preschool classroom.

Research Questions

The main research question for this study was: What is the role of sociomathematical norms in a preschool classroom? This question generated the following subquestions:

a. How are sociomathematical norms constructed in a linguistically diverse classroom?

b. What are the obstacles to the co-construction of these norms?

c. What conversation strategies does the teacher use to support the co-construction of sociomathematical norms?

Limitations

This study relied on interviews and observations as sources of data. Because of the qualitative nature of the data, findings are limited in generalizability. More research is required in this area before generalizations can be made. Furthermore, the classroom under examination was part of a private school. Therefore, the findings are not generalizable to public prekindergartens.

A serious limitation of this study was that I examined a classroom in the preschool where I work. The participating teacher was my colleague. Thus, bias and
conflict of interest were threats to the ethics and the trustworthiness of the study. I was bound ethically to report any wrongdoing of my colleague, even if I were moved to protect her. No wrongdoing did occur during my observations. However, it did fall upon me to honestly report some negative aspects of my colleague’s teaching style. I was bound as a researcher to report findings truthfully, even if these findings represented my participants negatively. Perhaps most importantly, the ethnographic nature of the study was compromised in that I shared some feedback with the teacher, which shaped her subsequent plans for instruction. I describe this influence I had on the participants in the chapters to come.

I practiced reflexivity throughout the study by maintaining a field journal and sharing my findings with the participants. Ongoing self-reflection, as well as ongoing and honest dialogue with the participants, helped protect against bias and conflict of interest. These practices enabled me to paint a fuller picture of the situations under study, so that seemingly negative behaviors on the parts of my participants can be understood in context. Dialogue enabled me to offer descriptions with the participants’ perspectives on the situations I had observed. In this way, data and analysis are offered in the chapters to come in a way that is both fair and comprehensive.

Assumptions

A number of assumptions underpinned the current study. The methodological design was built upon the theoretical and conceptual framework outlined in the preceding sections. I assumed that children and teachers actively co-construct norms for conversation and reasoning as part of a larger process of cultural reproduction (Yackel &
Cobb, 1996). Specifically, I assumed that these teachers and students reproduce larger mathematical culture in a unique way in their own preschool classrooms. I assumed that this reproduction of culture would be visible and audible during my visits such that I could relay the story to the readers of this study.

The primary assumption that underpinned the above assumptions is that young children internalize cultural ways of reasoning as they acquire domain-specific language. I assumed that observations of teachers and children during group conversations and interviews with both of these parties would enable me to glimpse the mechanics underlying this internalization process. Having made a purposeful sample (selection of one pro conversation preschool teacher), I assumed, albeit wrongly, that this teacher truly was exemplary in his or her ability to teach children to reason about mathematical concepts through conversation. Finally, I assumed that this particular study would fill the dire need for research on mathematical conversations at the preschool level.
CHAPTER 2
LITERATURE REVIEW

For centuries, teachers around the world have done most of the talking in mathematics classrooms (Cazden, 2001). Students were required to sit quietly and listen, never speaking unless spoken to. When their teacher did call upon them, they were required to give a short answer, usually by rote and/or from memory. The teacher would then tell the child whether or not he or she had given the right answer. This Initiation-Response-Evaluation format (Mehan, 1979; Sinclair & Coulthard, 1975) was observable in classrooms around the world. Traditional teachers rarely invited students to discuss the reasoning they had used to arrive at an answer. Even recent research shows that conversations in preschool classrooms usually consist of teacher directives followed by single-word responses on the part of children (Wasik, Bond, Hindman, & Jusczyk, 2007). In the United States, this transmission model (Resnick & Nelson-Le Gall, 1987) of instruction has failed and continues to fail to ensure students' conceptual understanding of mathematical concepts (Mercer, 1995; National Center for Education Statistics, 2017). Students learn procedures and rules, but often fail to grasp ideas conceptually in order to manipulate numbers in creative ways. This school failure does not bode well for students’ progress in real-world workplaces, where innovation and mathematical creativity are a requirement (Darling-Hammond, 2010), for students must engage in conversation to internalize knowledge of mathematical concepts (Yackel & Cobb, 1996).
Little research exists on this topic (Klibanoff, Levine, Huttenlocher, Hedges, & Vasilyeva, 2006). However, research from numerous fields adequately framed my current study. For example, studies from the fields of cognitive science (Barth, La Mont, Lipton, & Spelke, 2005; Clements & Sarama, 2007; Huttenlocher, Jordan, & Levine, 1994; Klein & Bisanz, 2000; Lee & Ginsburg, 2007b; Mix, 1999, 2002; Mix, Huttenlocher, & Levine, 1996) and neuroscience (Cantlon, Brannon, Carter, & Pelphrey, 2006; Dehaene, Piazza, Pinel, & Cohen, 2003; Rivera, Reiss, Eckert, & Menon, 2005) suggest that preschool children are neurologically and psychologically equipped to reason in complex ways about mathematical concepts. Studies from linguistics and education research demonstrate the importance of student fluency in academic language (Schleppegrell, 2001, 2004; Walshaw & Anthony, 2008).

Because of the importance of academic language fluency for academic success, students need exposure to this language as soon as possible. Studies from the field of education also demonstrate that K-12 children are capable of engaging in sophisticated mathematical conversations (Cobb, Stephan, McClain, & Gravemeijer, 2001; Yackel & Cobb, 1996). Finally, research on teacher conversation strategies suggest that some teachers have successfully engaged at least K-12 students in productive mathematical conversation (Chapin & O’Connor, 2007; Franke et al., 2009; Temple & Doerr, 2012; Wood, 1989). Rigorous qualitative research offers examples of rich mathematical conversations that take place at the kindergarten, first-, and second-grade level (Cobb & Bowers, 1999; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996). This research must now be
extended to investigate conversations at the preschool age. What features characterize preschool-level mathematical conversations? The current study answered this question among others.

Only one study investigated math talk for preschool classrooms (Klibanoff et al., 2006). None explored verbal mathematical discourse in a preschool classroom in a holistic qualitative fashion. This lack of research is unfortunate in light of findings that conversational focus leads to academic achievement for K-12 students (Doherty, Hilberg, Pinal, & Tharp, 2003). If the same is true for preschoolers, the lack of research in this area could result in many lost opportunities for the improvement of instruction.

This study began to answer the question: what does successful mathematics conversation look like at the preschool level? Research shows that teachers generally do not engage students in rich conversation in any content area (Gest, Holland-Coviello, Welsh, Eicher-Catt, & Gill, 2006; Massey, 2004). I purposefully sampled a preschool teacher who does teach math and who I believed engaged children in rich conversation about mathematical topics. This study resulted in a qualitative portrait of one teacher’s journey as she sought to engage her preschool students in developmentally appropriate conversation. The findings will serve to encourage other teachers in their classroom practice and to open further lines of inquiry for researchers interested in how to preschool teachers can support mathematical discourse in the classroom.

A Change in Epistemology for Mathematics Instruction

Recent research shows that mathematics classrooms should be discourse-rich (Gee, 1996; Schleppegrell, 2004; Temple & Doerr, 2012). Not just the teacher, but the
students should be talking, and in increasingly complex ways (Franke et al., 2009). These researchers contend that the transmission model should be replaced by a different model of teaching and learning in which students actively construct ways of reasoning and speaking mathematically. In the constructivist model (National Council of Teachers of Mathematics [NCTM], 2000), students are encouraged to speak up and converse with the teacher and with each other. I attempted to select a teacher who would engage in constructivist-style pedagogy. A great departure from the transmission style of instruction, the constructivist model encourages discourse-rich instruction. Teaching becomes a complex task of socialization and not a simple one of information delivery.

Constructivism is a product of the work of both Vygotsky (1934/1978, 1934/1986) and Piaget (1951, 1952). As I discussed in my theoretical and conceptual framework, Lave and Wenger’s (1991) situated learning theory also reflects a constructivist philosophy. As legitimate participants in the mathematics community of practice, students learn to politely argue with one another the way true mathematicians do (Yackel & Cobb, 1996). In this new model of mathematics instruction, students do much of the talking—and not just any kind of talk. They must provide explanations for their answers and press for deeper understanding of mathematical problems. Giving answers alone is not related to positive achievement outcomes (Webb & Palinscar, 1996). Students who provide explanations for their answers experience better achievement outcomes, even when researchers control for their previous achievements. When these explanations are detailed and precise, fellow classmates can also benefit from exposure to mathematical reasoning.
Research on talk-intensive mathematics instruction has proliferated over the last three decades (Walshaw & Anthony, 2008). The maturation of research on talk-intensive mathematics instruction can be explained in part by the publication and dissemination of the NCTM standards (1989, 2000). In standards first published in 1989, the NCTM introduced the unorthodox idea that students, not teachers, should be at the center of mathematical conversations. These standards set the bar in the areas of curriculum, teaching, and assessment. They provided guidance to those making important decisions regarding math instruction for children at the Kindergarten-12 level. NCTM also published the revised Principles and Standards for School Mathematics (PSSM) in 2000 to include preschool. Further, a 2010 publication on mathematics education for young children was a joint position paper by the National Association for the Education of Young Children (NAEYC) and NCTM. The authors of this paper suggested that preschool-age children, too, are ready to engage in conversation about numbers and operations. It is worth noting that the research on young children in these papers stand to be updated. My study served this purpose.

Important Themes in Mathematical Discourse Research

Important themes in recent research on mathematical communication have included sociomathematical norms (Lopez & Allal, 2007; Yackel & Cobb, 1996), academic language (van Kleeck, 2014) linguistic differences (Barletta, 2008; Purpura & Reid, 2016), and teacher conversation strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012). Although overlapping, I have not found research that explored the relationships between these major topics. I will synthesize these topics in a novel way in
this paper. The topic of linguistic differences most recently includes the concept of academic language (van Kleeck, 2014). As I will explain in the following sections, student linguistic difference can enable or impede the ability to access mathematical conversations. Any study of mathematical conversation should address students’ linguistic differences. In this study, I identified two students who were in the linguistic minority in Chen’s classroom and analyzed data on their participation or lack of participation in premath and mathematical conversations.

The research on sociomathematical norms served as a conceptual framework for understanding linguistic differences. Yackel and Cobb’s (1996) sociomathematical model describes mathematics classrooms as microcommunities in which sociomathematical norms are constituted through language. Sociomathematical norms are norms for reasoning that are manifested through language. Although research on mathematical language (van Kleeck, 2014) refers to a unitary register, the reality is that teachers and students in each classroom manifest mathematical language in unique ways (Morgan, 1998; Temple & Doerr, 2012; van Kleeck, 2014; Yackel & Cobb, 1996). In this sense, language can be considered to be pluralistic. Sociomathematical norms are norms for reasoning and speaking mathematically. Mathematical language contains norms for speaking mathematically. Thus, the two concepts overlap. This overlap has not been considered in the relevant literature. However, the overlap became apparent in this review of the literature.

Classroom microcommunities serve as the contexts in which mathematical language is constituted simultaneously with sociomathematical norms. As teachers and
students co-construct these mental apparatus, student linguistic difference becomes apparent. Some students gain easier access to mathematical conversations, shaping the norms such that they benefit themselves more than the linguistic minority students. The result is that the emerging classroom culture benefits some students more than other students. This fact has not been noted in the relevant literature, but through synthesis of the literature, it is apparent.

Linguistic differences may have a profound upon the emergence of sociomathematical norms and academic language in the classroom microculture that I studied. The researchers of studies on sociomathematical norms (Lopez & Allal, 2007; Yackel & Cobb; 1996) and researchers of studies on academic language (Halliday, 1978; Pimm, 1987; van Kleeck, 2014) fail to make an explicit connection between their work on mathematical conversations and the research on linguistic differences (Barletta, 2008; Purpura & Reid, 2016). However, as I synthesized their work hypothetically, it appeared that linguistic differences between students could profoundly influence their ultimate success in learning how to engage in mathematical conversations.

Barletta (2008) did find that students who speak different dialects or registers than the teacher experience less access to classroom conversations. Researchers have called these students linguistic minorities (Heller, 2006). Teachers favor those children who are already fluent in academic language by conversing with them more. It logically follows that, in light of Yackel and Cobb’s findings (1996), linguistic majority students will enjoy more opportunity to co-construct sociomathematical norms. This study verified that this was the case in one preschool classroom. I investigated the mechanisms that underlay the
process by which some students may be more linguistically advantaged in mathematical conversations.

Cobb and colleagues investigated math talk at the early childhood level (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996). These researchers worked in kindergarten, first-grade, and second-grade classrooms. Barletta (2008) also studied older children. However, van Kleeck investigated the math talk of older children (2006, 2008, 2014). None of these researchers addressed math talk for preschool-age children. Furthermore, the above research needs updating. This study served to bring research on sociomathematical norms, mathematical language, and linguistic differences up to date, and also to explore the interaction of these concepts in the context of a preschool classroom. However, it is important to note that the ethnographic design of the current study required that the data collection remain somewhat open-ended. Thus, the data collected revealed that other themes were at times more important than those I previously outlined. These themes, which emerged out of the preliminary literature review, served as a guide where they proved relevant to the data.

The final theme from the relevant literature I reviewed was teacher conversation strategies. Teachers who plan for productive conversation can use specific strategies in order to extend mathematical conversation and include more students in the conversation. Thus, it is possible for teachers to mitigate the inequitable pattern described above. Through the use of conversational tools, teachers can make sure that both linguistic majority and linguistic minority students enjoy a chance to shape
classroom norms for thinking and speaking mathematically. The result is a classroom microculture that reflects the reasoning styles of all the students.

Neuroscience Findings and Children’s Capacity for Numerical Reasoning

The pioneer French neuroscience researcher, Dehaene (1989, 2001, 2011) and colleagues (Dehaene et al., 2003; Nieder & Dehaene, 2009) made extensive study of the neural mediation of number sense. Dehaene (2001) explained, “Number is a fundamental parameter by which we make sense of the world surrounding us” (p. 16). All languages, he pointed out, include words for numbers, and people of all cultures across time and place reason about numbers and operations. This study shed light on what researchers in many fields call number sense in the context of one preschool classroom. Number sense underpins later strands of mathematical instruction, such as advanced arithmetic, algebra, and geometry (NAEYC, 2010). Thus, the development of number sense is important to understand for the purposes of early childhood mathematics instruction.

In his landmark book, The Number Sense, Dehaene (2011) argued for a composite nature of number sense. In his model, mathematical talk plays an important role in the development of number sense. In this composite view, number sense is not mediated by one area of the brain, but rather three areas that work in concert: the innate or approximate system, the verbal system, and the visual system. Dehaene suggested that strong number sense results from linkages between these three distinct areas, so that in a mature mind, the three work in concert.

Dehaene’s (2011) view of number sense validates both the Piagetian and Vygotskian views of child development. The Piagetian view is that learning trajectories
are biological and innate, while the Vygotskian view is that development is pushed forward and developed to maturity with experience. In Dehaene’s account of number sense, both views are reconciled. The areas of the brain associated with number sense exist prior to any experience, and yet experience making connections between these disparate areas can result in stronger number sense. Both biology and experience contribute to the mature development of number sense.

Although Dehaene (1989, 2001, 2011) receives credit for popularizing and making many advances in the field of numerical cognition, he was not the first to find that specific areas of the brain were involved in the development of number sense. The idea was suggested first in early studies on brain lesions (Gerstmann, 1940; Henschen, 1919). Scientists noticed that when areas of the parietal lobe were damaged by lesions, patients lost the ability to calculate or reason numerically. As technology allowed the field of neuroscience to flourish, scientists used positron emission tomography (PET) (Zago et al., 2001) and later functional magnetic resonance imaging (fMRI) (Dehaene et al., 2003) technology to observe the activation of the parietal lobe, precentral cortex, and prefrontal cortex during calculation.

Later research using refined fMRI methods confirmed that numerical representations resided in the parietal lobe (Cantlon et al., 2006; Dehaene et al., 2003). Dehaene and colleagues constructed a different model for understanding numerical cognition than did Cantlon and colleagues. Dehaene and colleagues argued for a composite understanding of number sense in which three areas of the brain are involved and become increasingly interconnected as number sense develops. By contrast, Cantlon
and colleagues found that one area of the brain is responsible for numerical
cognition. Rivera and colleagues (2005) found yet different areas to be involved in
numerical cognition. The discrepancies between these three models are interesting to
note, but not necessarily relevant for the current argument. It is relevant to note that in all
three models, children are shown to possess cognitive architecture that makes them ready
to develop numerical reasoning. Further, all groups of researchers found that numerical
reasoning takes place mainly or partially within the parietal lobe. It is clear that young
children possess cognitive machinery necessary for more sophisticated number sense
work than previously assumed.

In Dehaene and colleagues’ (2003) synthesis of nine fMRI contrast studies, they
found that the areas of the brain involved in number sense activities include the bilateral
intraparietal sulcus, a region of the left angular gyrus, and a posterior superior parietal
system. In this study, the researchers looked at the fMRI scans of participants engaged in
a variety of contexts. They compared and contrasted approximately 36 scans from each
participant in order to determine which activities resulted in more parietal lobe
activation. For example, they compared participants’ exact addition of one-digit numbers
versus approximate addition of one-digit numbers. Dehaene et al. (2003) found that
approximate addition took place in the area of the brain associated with innate, primitive
number sense, while exact addition did not. In another example, the researchers
compared scans of participants’ letter naming with their subtraction of one-digit
numbers. They found, again predictably, that subtraction of one-digit numbers activated
the area associated with innate number sense while letter naming did not.
Again, this study suggests a composite view of how numbers are represented in the brain. In *The Number Sense* (2011), Dehaene suggested that four lines of evidence point to the existence of a biologically determined aptitude for the development of number sense in preschool-age children. The first is evidence of the evolutionary precursor of number sense in animals (Hauser, Carey, & Hauser, 2000; Roberts, 2010). The second is evidence of early arithmetic abilities in infants (Starr, Libertus, & Brannon, 2013). The third is the evidence from neuroscience research of a neural substrate dedicated to number sense in the parietal lobe (Cantlon et al., 2006; Dehaene et al., 2003). However, more research is necessary to create a complete picture of how number sense develops for preschool-age children. For the moment, the conclusion of these researchers (Dehaene et al., 2003) point to the critical importance of verbal communication, written notation, and possibly objects (which are also visible) for the purposes of teaching mathematics.

In Dehaene’s triple-code model for understanding numerical cognition (Dehaene & Cohen, 1995; Dehaene & Mehler, 1992; Dehaene et al., 2003), numbers are represented first in the bilateral intraparietal system. This system is associated with the innate, approximate system for perceiving number that animals and babies possess before any formal experience or instruction in mathematics. Subsequently, numbers are represented in the verbal and visual systems. The verbal number processing takes place in the left angular gyrus. Visual processing of number takes place in the posterior superior parietal system. It is important to note that in this model, different types of numerical representation take place in different areas of the brain. However, Dehaene
(2011) argued, as children make cognitive connections between different forms of numerical representation, the areas of the brain begin to be activated together during different types of problem-solving activities. This intermodal activation is a sign of advanced number sense (Dehaene et al., 2003).

The triple-code model (Dehaene & Cohen, 1995; Dehaene & Mehler, 1992; Dehaene et al., 2003) offers exciting ideas for the field of mathematics education research. If teachers are aware of children’s capacities in all three systems of numerical representation, they may better capitalize upon them. They may also ensure that children make cognitive connections between these systems of representation in order to develop more powerful number sense (Dehaene, 2011). This research suggests that teachers can assist children by helping them see and make connections between the analog, verbal, and visual systems for representing number. Some subsequent researchers problematized the triple-code model proposed by Dehaene and colleagues (Cantlon et al., 2006). Still, the discovery that young children do possess areas of the brain dedicated to the development of number sense suggests that they are ready for more rigorous instruction in numbers and operations than previously assumed (Piaget, 1951, 1952).

Dehaene’s triple-code model (Dehaene & Cohen, 1995; Dehaene & Mehler, 1992; Dehaene et al., 2003) also suggests that verbal communication could be a critical component of classroom interaction. It would logically follow from this theory that conversation between teacher and student and between students would be very important for making the cognitive connections Dehaene and colleagues claim are so important. However, these researchers limited their studies to number words and did not
extend study to more complex conversation. Further, these researchers limited their study of the visual component to the sight of written numbers, and did not extend their study to participants’ viewing of mathematical objects and/or manipulatives. Thus, the triple-code model is promising but still limited. The model suggests that conversation and the use of manipulatives could be important for mathematical learning, but it does not prove it.

In his most recent study relevant to the topic of number sense, Dehaene and colleagues (2003) examined images of brain activation while participant adults engaged in numerical activities with nonmathematical activities in the context of the fMRI contrast paradigm. Mathematical activities included comparison of one-digit numbers, subtraction of one-digit numbers, and multiplication of one-digit numbers (Dehaene et al., 2003). The participants took part in 18 different numerical activities. The experimenters then used software to aggregate the two-dimensional images of brain activation for each activity. In this way, they created a three-dimensional image of a hypothetical brain. These findings showed that analog, verbal, and visual activities corresponded with neural activations in the areas of the brain indicated in the triple-code model. These findings thus supported Dehaene’s previously articulated model (Dehaene & Cohen, 1995; Dehaene & Mehler, 1992; Dehaene et al., 2003). Dehaene’s work is relevant for the current study because the areas of the brain identified as being associated with numerical processing are present in the brains of both children and adults.

One could criticize the Dehaene et al. (2003) study by pointing out that comparison of one-digit numbers is not necessarily representative of analogical code
activity. Nor is it clear that multiplication is a verbal code activity. Dehaene and colleagues examined activities that they believed representative of the analog, verbal, and visual codes, but their selections were based upon ambiguous and uncertain assumptions. In fact, the triple-code model has not yet been rigorously proven to have explanatory power. Before discarding the triple-code model, it is important to note that Dehaene articulated the model as a result of research he conducted over the course of many decades (Dehaene, 1989; Dehaene, Bossini, & Giraux, 1993; Dehaene, Dupoux, & Mehler, 1990; Dehaene & Mehler, 1992). Although his assumptions are not always as transparent as they should be for those outside of his circle of research colleagues, his model may be better empirically validated in time.

Fortunately, for this study, a subsequent group of researchers studied further fMRI data to determine how development of number sense is mediated neurologically. Cantlon and colleagues (2006) also found that number sense activities were mediated by neural networks in the parietal lobe. While Dehaene worked with adults and merely extrapolated his findings to make assumptions about young children, Cantlon and colleagues studied two groups: four-year-old children and adults. Because Cantlon and colleagues studied children and not just adults, their study is even more relevant for this study. It is also more recent and included cutting edge fMRI imaging technology. Cantlon and colleagues (2006) studied 12 healthy young adults and 8 healthy four-year-olds.

The children and adults received the same tasks (Cantlon et al., 2006). They were to lie in an fMRI scanner and look at screens showing configurations of blue
objects. These objects varied in number from 16 to 32, and the elements varied in shape: circles, squares, or triangles. The participants each viewed 238 images. The screens were shown in pairs. Sometimes the pairs showed the same numerosity, shape, and configuration of shapes. Other times, the images differed. The researchers observed the brain activity of the participants particularly when the images differed in numerosity. Unfortunately, the large number of slides was itself a limitation of the study. Four-year-old children may have become tired of thinking about the images after seeing so many. Numerical cognition may have ceased to be at play after viewing a certain number of screens (Cantlon et al. 2006).

Cantlon and colleagues (2006) found that whenever the pairs of screens differed in numerosity, the intraparietal sulcus (IPS) was activated. This was true for both four-year-olds and for adults. This finding led the researchers to assume that the IPS was the area of the brain principally involved in numerical cognition. This study offered empirical evidence for Dehaene’s (2011) theory that infants, as well as small children, possessed cognitive equipment necessary for the development of number sense. Cantlon and colleagues (2006) did not offer as much variety in their tasks as did Dehaene and colleagues (2003). Further while Dehaene and colleagues (2003) found that three areas of the brain were involved in numerical cognition, Cantlon and colleagues (2006) found that one area of the brain was particularly involved in these activities: the IPS.

The differences in the findings of these two groups of researchers (Cantlon et al., 2006; Dehaene et al., 2003) were relevant for this study in that Dehaene and colleagues argued for an area of the brain particularly dedicated to the verbal communication piece
of number sense. Otherwise, both of these groups of neuroscience researchers offer the idea that biological machinery ensured that preschool-age children were ready for instruction in numbers and numerical operations. A broad question now lingers: how can teachers take advantage of this readiness in order to help children develop more sophisticated number sense? On this subject, there is a paucity of research.

In my extensive searches, I only identified two neuroscience studies that investigated the numerical cognition of children, and only Cantlon and colleagues (2006) investigated the cognitive machinery of preschool-age children. Another study relevant to my investigation was that of Rivera and colleagues (2005). This study included 17 participants, which included older children as well as adults, ranging in ages between eight and nineteen, with a mean age of thirteen. It did not include preschool-age children. However, this study does offer an exciting look into the continuity of numerical cognition from childhood into adulthood, suggesting that children are just as biologically able to reason numerically as adults (Rivera et al., 2005).

Rivera and colleagues (2005) investigated changes in the neural mediation of arithmetical activities. This study included 17 healthy, right-handed participants whose ages ranged from eight to nineteen. The mean age of these participants was thirteen. The researchers investigated anatomical changes in the brain related to age using fMRI images. Using fMRI images, 18 axial slices were created covering the whole brain. The experiment consisted of seven phases. In the first phase, participants were at rest for 30 seconds. During this phase, participants simply looked at a blank screen. Subsequently, they experienced six alternating experimental and control phrases
that lasted for 30 seconds each. During the experimental phases, participants viewed completed two-operand addition or subtraction problems. Some of these equations included the correct answer while others included an incorrect answer. Participants were instructed to press a button when the answer to the equation was correct (Rivera et al., 2005).

During these seven phases, 18 axial slices of the whole brain were imaged. In the analysis of these images, it became clear that, cognitively speaking, younger participants depended more on working memory, attentional resources, and declarative and procedural memory. By contrast, older participants used less of these resources. The images of these older participants also showed an increased specialization of the left posterior parietal cortex (Rivera et al., 2005).

These findings suggested that as children matured neurologically, they experienced increased functional specialization of the left posterior parietal cortex. At the same time, they became less dependent upon the prefrontal cortex (Rivera et al., 2005). What did this mean? Reduced dependence upon the prefrontal cortex suggested that less assistance from working memory, attentional resources, and declarative and procedural memory was needed. Solution of arithmetic problems became more automatic and less effortful. Further, it was apparent that neural pruning occurred so that less of the brain was involved in completing the same tasks. Solution of arithmetic problems was thus more efficient (Rivera et al., 2005).

A major limitation of this study was that Rivera et al. (2005) did not examine what kinds of experiences or interventions occurred to assist the process of increasing
automaticity to occur. The data clearly showed that numerical cognition becomes more automatic with age, but what kinds of experiences best help accelerate this change? In my searches, I found no studies that helped explain this process. My study, which leaned upon the older theoretical work by Vygotsky, helped close this gap in recent research. Vygotsky found that when children converse with adults about mathematical problems, they appear to internalize these conversations in order to reason more effectively about solution of the problems. Just like the participants in the experiments Vygotsky conducted, the participants in Rivera and colleagues’ (2005) study appear to have internalized what once was effortful problem solving. Problem solving for these older participants was immediate and even automatic.

Vygotsky’s studies and theoretical work suggested that children internalize high-quality conversation about numbers and arithmetic. What occurred on the level of conversation was internalized to become the child’s inner reasoning. Perhaps the participants in the study conducted by Rivera and colleagues (2005) also internalized math talk when they were of preschool age. However, this question was not addressed by Rivera and colleagues or any other studies that I have found. Cobb and colleagues did in fact research this question at the early childhood level and found that kindergarten, first-grade, and second-grade children internalized high-quality conversation about math conversation to become better at reasoning mathematically. However, Cobb and colleagues did not investigate this topic at the preschool level. My study helped resolve this discrepancy.
These areas of the brain pinpointed by Rivera and colleagues (2005) did not overlap with the areas identified by Dehaene and colleagues (2003) or by Cantlon and colleagues (2006). In other words, the findings of Dehaene and colleagues (2003), Cantlon and colleagues, and Rivera and colleagues (2005) each pointed to different areas as responsible for mediating numerical reasoning. This discrepancy pointed to the need for further neuroscience research on the topic of children’s number sense.

Of the available neuroscience studies, these studies (Cantlon et al., 2006; Dehaene et al., 2003; Rivera et al., 2005) were those that were most relevant to the current questions. Only two pertained to children (Cantlon et al., 2006; Rivera et al., 2005) and only one pertained to preschool-age children (Cantlon et al., 2006). This selection once again reveals that early childhood mathematics does not currently receive a great deal of serious attention. In light of Cantlon and colleagues’ findings, preschool-age children possess the same cognitive machinery that adults do when it comes to working with numbers. This finding suggests that researchers should take early childhood mathematics seriously. Neuroscientists do not agree on which areas of the brain contribute to the numerical cognition of young children, or even of older children and adults. More studies are required in this field to understand the brain of the young child engaged in developing number sense. Further, educational research is required to understand what educational activities will assist young children in developing number sense.

Mathematical Cognition and Development of Number Sense

Current neuroscience research underscores the keen mathematical abilities of the young child (Cantlon et al., 2006). However, research leading up to the last two decades
had already used cognitive models to explore the abilities as well as the limitations of young children as they solve mathematical problems. This somewhat older research revealed that, depending on context, preschool age children of three, four, and five years old were able to represent both small (Mix, 1999, 2002; Mix et al., 1996) and large numbers (Barth et al., 2005), as well as perform operations on them (Huttenlocher et al., 1994; Klein & Bisanz, 2000). Some more recent cognitive research (Barth et al., 2005) verifies these older findings, demonstrating again that young children possess considerable ability to think abstractly about number and to perform numerical operations even before formal instruction begins.

These studies (Barth et al., 2005; Huttenlocher et al., 1994; Klein & Bisanz, 2000; Mix, 1999, 2002; Mix et al., 1996) demonstrated that young children could perform tasks that required mental representation of number. These studies came long before as well as after the current neuroscience interest in the topic. In the cognitive science studies, children are said to represent number mentally (Mix et al., 1996) in order to solve problems such as matching sets for numerical equivalence (Mix, 1999). It is assumed, children must represent numbers in their working memory (Klein & Bisanz, 2000) in order to manipulate them and successfully solve problems. In the following synthesis of the cognitive science research on children’s numerical representations, I outline the studies most relevant for the interpretation of my study.

In reading this synthesis, it is important to note how the findings of each study challenge the Piagetian paradigm. The children in the following studies do in fact conserve numbers, even if there are some limitations to their abilities (Mix, 1999; Mix et
al., 1996). The most interesting findings come out of the 2002 study of Mix’s own son. This is the only study I found in which the child’s abilities were investigated in a naturalistic setting (the child’s own home). In his home setting, three-year-old Spencer demonstrated an understanding of number that children in laboratory settings did not seem to possess. In fact, when Spencer himself performed mathematical tasks in the laboratory, his abilities appeared worse than they did when he approached similar tasks in his home context. This important study suggests that research on children’s mathematical abilities still requires a great deal more study. Successes in laboratory settings notwithstanding, children may possess far more mathematical abilities than researchers know. In fact, the research community will not know for sure just how capable children are until they are further examined in naturalistic, social settings. My study examined children’s mathematical development in just such a naturalistic setting.

Kelly Mix is a prominent researcher on the mathematical abilities of preschool-age children. In the late 1990s and early 2000s, Mix and colleagues conducted a series of studies on early childhood number sense. In these studies, children’s abilities to grasp and manipulate number were measured in terms of how well children could compare small and large numbers of items as well as add and subtract from these quantities. Mix’s (1999) study on numerical equivalence was an important follow-up to a previous (Mix et al., 1996) study. The first study investigated three- and four-year-olds’ abilities to match numbers of sounds to numbers of visual objects. Mix et al. (1996) found that three-year-olds were unable to match these items successfully. However, four-year-olds were successful in matching auditory with visual items. Mix et al. (1996)
found that age was not necessarily the only deciding factor, but also the mastery of counting language. This finding is relevant for my study because of the implication that mastery of mathematics language may lead to better ability to solve mathematics problems. Mix et al. (1996) only looked at counting language. My study deeply explored the manner in which mathematical language shaped mathematical reasoning and thus mathematical competence, therefore, continuing where Mix et al. (1996) discontinued. Mix et al.’s (1996) study also suggested that four-year-olds are more ready to grasp numbers than are three-year-olds, a justification for my study’s focus on four-year-old preschool students.

In a subsequent study, Mix (1999) explored the questions of the 1996 study in even greater detail. In Mix et al.’s (1996) experiment, auditory stimuli were presented to children before visual stimuli. In the next study, Mix (1999) explored whether the sequential nature of the experiment made the task more difficult. In order to answer this question, Mix presented some stimuli simultaneously and others sequentially. Mix also divided children into more refined age groups in order to better examine the abilities of each group. The results of this study were particularly relevant for the current study.

Mix (1999) investigated whether three- to four and a half-year-olds could judge numerical equivalence involving simultaneously, as well as sequentially presented stimuli. This question is important for any study of young children’s numerical understandings because children must represent quantity abstractly in their memory in order to compare quantities presented sequentially. Mix (1999) found that children who could count proficiently were more able to find numerical equivalence between stimuli
that were presented sequentially. This study thus presented more proof that mastery of counting language assists children in forming abstract representation of numbers.

Over the course of three separate experiments, Mix (1999) presented children with three types of stimuli: black dots on white cards, flashes of light, and a stuffed animal that the experimenter made to jump. Mix reasoned that these three types of stimuli represented visual objects (dots), nonobject events (lights flashing behind a screen), and actions (stuffed animal jumps). Mix found that children who could count performed better at all of the tasks. Children also performed better at every task as they got older. However, children could have been developing better mathematical language as they got older. This development could also have been assisting them in performing better on the tasks. Mix (1999) did not explore this possibility as I did in the current study.

It was interesting that the child’s ability to count was even more important for comparisons of quantities of light flashes and puppet jumps, which were by nature sequential and not static like the black dots. A child’s ability to count predicted better ability to evaluate the numerosity of these items, again lending credence to the current study. First, it is encouraging that such young children can think abstractly enough about number to match numerosities of sets in more than one modality. Furthermore, because counting seems to lead to success in these mathematical activities, it may be considered that further exposure to formal mathematical language can lead to greater ability for young children to solve math problems. My study highlighted the assistance that
mathematical language offered children in representing number and reasoning about number.

To understand the significance of Mix’s (1999) study for this study, it is necessary to take a closer look at the methods and findings. I review Mix’s study carefully because of the strong evidence it offers that children age four and over are able to tackle difficult mathematical problems. Sixty-four children participated in the first experiment, 48 in the second, and 48 in the third. In all three experiments, children were divided into three groups so that age differences could be examined as a mediating factor in task performance. In each experiment, the three groups consisted of three and a half year olds, four year olds, and four and a half year olds. All of the participants were drawn from preschools serving a predominantly White, middle-class population. There was an equal number of girls and boys. In each experiment, children took part in matching tasks as well as counting tasks. Children took part in counting tasks so that counting ability could be examined as a mediating factor in performance on matching tasks. In each trial of each experiment, the experimenter interviewed an individual child. Children were familiarized with the activities before experimental trials began to ensure that they understood how the activity worked.

In the first experiment, children matched cards marked with black dots with other cards marked with black dots. The dots on the cards numbered from one to five. The experimenter first showed the child a card marked with an array of black dots. Then the experimenter showed the child two response cards to choose from. One card showed the
same number as the target card, in a different arrangement, and the foil card showed a
different number of black dots (Mix, 1999).

In simultaneous presentations, the child was free to compare the target set with
the response cards before choosing the equivalent set. In the sequential presentation,
children were showed the card bearing the target set. Next, this card was covered with a
box before the choice cards were shown. These were the same procedures using in Mix
et al.’s (1996) study during the visual-visual condition. In this study, children were given
12 test trials per condition (Mix, 1999).

One purpose of the experiment was to determine whether sequential presentation
might add further difficulty to the visual-visual procedure described in Mix et al.’s (1996)
study. The results of the ANOVA model suggested that sequential presentation did
indeed add difficulty for the participating children. Children’s performance on the
sequential task was significantly worse than on the simultaneous condition. Another
important finding was that the three and a half year olds performed significantly worse on
both tasks than the four year olds and four and a half year olds. Only four year olds
performed as well on the sequential task as they did on the simultaneous task. This
finding is significant for my study because of the implication that four year olds may
grasp number and operations much better than three year olds can. To perform well on
the sequential study, some ability to represent number mentally was necessary. In order
to hold the number in working memory (Klein & Bisanz, 2000), it was necessary to
represent it mentally in some way. This finding suggested that slightly older children
may have much greater potential to talk about and learn about number and operations.
For the remainder of the data analysis of experiment one, children were divided into groups based on how well they counted during the counting tasks. Children who were minimally proficient counters performed above chance on both matching tasks. In fact, it emerged that children who were deemed noncounters only performed at chance on the simultaneous task. Counting was in fact such a powerful aid in performing these matching tasks that only children who could count performed above chance on the sequential tasks (Mix, 1999).

The last experiment revealed that proficient counters were more able judges of numerical equivalence of objects. In this experiment, the target set types included flashes of light and the jumps of a stuffed animal. In each trial, the child chose the response card (the same cards from the previous experiment) whose numerosity matched that of the target set, just as in the previous experiment. However, in this experiment, Mix (1999) examined whether children were able to enumerate flashes of light and jumps of a puppet as well as they could objects (dots on the card). Half the children were shown the flashes of light and half were shown the stuffed animal jumps.

For the light flash matching task, children were shown flashes of light from behind a screen. Thus, no object was visible. Next, the child was shown two cards from the first experiment and asked which card matched the flashes of light in numerosity. For the task involving the stuffed animal jumps, the experimenter picked up the stuffed animal and made it jump up and down on the task between one and five times. The child then chose one of two response cards in order to match the numerosity of the jumps. ANOVA scores showed that event-matching scores were not significantly
different for light flashes or for stuffed animal jumps. It appeared that children were able to judge numerical equivalence just as well for one as for the other. Because one involved an object and the other an event not involving an object, the experimenter concluded that children were equally capable of discerning number for both (Mix, 1999).

However, children did not match the above events or actions as well as they did the objects in the first experiment. The ANOVA procedure revealed that children matched objects in the first experiment significantly more successfully than they did the light flashes and stuffed animal jumps. Furthermore, an age effect was found. The scores of the three and a half year olds were lower than the scores of the four year olds and four and half year olds on the second experiment. Mix (1999) found that children were able to successfully match sequential-object sets at a younger age than sequential-event sets. Similar findings were verified in the study of Mix’s own son (Mix et al., 2002).

Counting helped children in experiment two even more than it did in experiment one, suggesting that the problems posed in experiment two (judging numerical equivalences using flashes of light and stuffed animal jumps) were more difficult for the participating children. However, mastery of counting language was beneficial enough for children that they were able to solve the more difficult mathematical problems posed in experiment two. Again, a limitation of the study was its failure to investigate how children’s mastery of other mathematical language may have assisted them in performing the tasks. In light of how significant mastery of counting language proved to be, an
investigation into the utility of other mathematical language for children’s performance is warranted (Mix, 1999).

Experiment three focused solely on how children could match stuffed animal jumps. However, this time, the child watched the experimenter make the stuffed animal jump, and then matched the numerosity of the jumps by making the stuffed animal jump him or herself. In the stuffed animal jumps condition, the experimenter made the animal jump between one and five times. Next, the child was to imitate the experimenter and make his or her own stuffed animal jump the same number of times. There were six trials following this procedure (Mix, 1999).

As shown in the comparison between results for experiments one and two, children scored higher on sequential matching for objects than they did on sequential matching for actions. Children of ages four and four and a half also performed better than children aged three, just as in experiments one and two. This finding once again suggests that children age four and above are more ready for formal mathematics tasks than are children age three and under. This experiment also confirmed that children who were more proficient counters were also better at accurately matching stuffed animal jumps regardless of their age (Mix, 1999).

I have given this study a careful review because of the strong evidence it offers that children aged four and over are able to tackle difficult mathematical problems. This study reveals that sequential matching tasks are more difficult than simultaneous-matching tasks (Mix et al., 1996). Furthermore, sequential-matching tasks involving events and actions are more difficult than those involving objects. In spite of these added
requirement for abstraction, children of four and four and a half are capable of performing above chance on these tasks. Another point of great interest in this study is the importance of counting for performance on difficult mathematical problems.

Finally, the study is interesting because it represents a revision of the classic Piagetian studies, especially in experiment one. Children in Piaget’s study were unable to recognize numerical equivalence when two arrays of coins were arranged differently. In Mix’s two relevant studies, four-year-old children recognize numerical equivalence with ease even when arrays of black dots were arranged differently. This discrepancy highlights the need for variations on classic studies. It could be that the verbal part of the presentation in Piaget’s earlier studies confused children so that their true mathematical ability was misrepresented.

Another study undertaken by Klein and Bisanz (2000) demonstrated how many unique problem-solving strategies four-year-old children can employ spontaneously. As in Mix’s (1996, 1999, 2000) studies, an opportunity was lost to hear from children about their justifications for choosing particular solution strategies. However, like those above, Klein and Bisanz’s (2000) study strongly suggested the need for the current investigation into children’s verbalizations of their thinking. Although Klein and Bisanz did not ask children to offer their reasoning, children spontaneously did so when in the process of using some particularly difficult strategies. It seemed that young children were ready and willing to talk about their solution strategies. Further, their thinking was diverse and full of possibility for conversation.
Klein and Bisanz (2000) investigated young children’s early development of number sense. These researchers pointed out that the problems that they offered students were quite difficult because the problems were arbitrary and out of social context. Like Mix’s (1996, 1999) studies, these tasks included matching and counting in a laboratory setting with only the experimenter/s standing by. However, these experiments also tested children on their ability to perform addition and subtraction mentally. These children once again performed above chance on the tasks offered like the four year olds in Mix’s studies.

Klein and Bisanz’s (2000) study involved 48 children whose median age was four years and four months. Experimenters engaged children in a “math game” during two fifteen minute sessions on separate days. Children were pretested for counting and matching abilities. One child was disqualified because he was unable to count to ten. Another five were disqualified because they failed to understand the matching task. These children were replaced with children who successfully passed the pretests.

The children and the experimenter each had 10 white chips. Using these white chips, the experimenter demonstrated two-term and three-term addition and subtraction problems. In a typical two-term problem, the child was shown an array of six white chips. After showing the child the array, a box was placed over the array. The experimenter reached under the box and added or removed a number of white chips, showing the disks to be added or removed to the child. After having seen both the array and the number added or removed from the quantity, the child was to “match” the array
now remaining by arranging his or her own chips in imitation. Finally, the experimenter showed the child the true remaining array (Klein & Bisanz, 2000).

The three-term sessions were similar except that the quantity of the array was changed twice. The child was shown the initial array. The box was placed over the array. A quantity was added or taken away—the child saw the chips to be added or taken away once more. Then a second quantity was added or subtracted, with the child’s full awareness. The child was to use his or her chips to match the array he or she believed to be under the box. Finally, the experimenter then showed the child if he or she had been correct (Klein & Bisanz, 2000).

Preschoolers, just like older children as well as adults, demonstrated a problem-size effect. Problems involving larger numbers were more difficult than problems involving smaller numbers. This conclusion successfully replicates findings by Huttenlocher et al. (1994). The researchers postulated that children’s difficulty in solving problems with larger numbers was due to working memory difficulty in holding larger numbers in the memory (Klein & Bisanz, 2000).

Klein and Bisanz (2000) found that children used quite a variety of procedures in order to solve three-term problems. The variety of solution strategies suggested that children would have enjoyed ample topics for conversation. Had social interaction and conversation been a part of this investigation, children may have learned how to compare and contrast their solution strategies for each other’s mutual benefit. However, Klein and Bisanz (2000) considered individuals in isolation and did not elicit any verbal explanation from the children as they employed their novel strategies in solving these problems. In
my searches, I found no study in which preschool-age children were encouraged to explain their thinking as they solved mathematical problems.

In one type of problem solution strategy, children added and/or subtracted chips to their arrays physically and in two separate steps. In other words, children relied on the physical imitation of the experimenter in adding or subtracting chips. This solution type was not the most typical. Klein and Bisanz (2000) coded these solution strategies as sequential. Older children were likely to use knowledge of number facts. In studies with older children, conversations often concerned which math facts these children applied to problems and how. However, it stands to reason that young children who relied on the use of manipulatives could also have verbalized their thought process in doing so.

Klein and Bisanz (2000) coded another type of solution strategy as associative. This type of solution strategy was also used infrequently. In this solution strategy, the child applied the second procedure described by the experimenter before the first one. The child effectively switched the order of procedures, thereby demonstrating knowledge that the order of procedures did not affect the outcome. Again, the child’s knowledge in this case could have served as subject of conversation. These conversations could have served to educate other children who did not possess this understanding. Children who used this strategy sometimes arrived at the correct answer. However, some children added or subtracted incorrectly. An effective teacher could have used both correct and incorrect answers as subject of productive conversation.

Inappropriate procedures were also applied infrequently in this experiment (Klein & Bisanz, 2000). In these attempts, the child subtracted what should have been added or
vice versa. Finally, the most frequent solution strategy was coded as covert. In this sophisticated solution style, children listened to the problem \((2 + 3 - 1)\) and reduced the problem to a single operation, simply adding two chips to the original two. This frequently-used style of solution revealed that children were more than capable of understanding and solving simple math equations. They were also able to solve part of the problem in their minds. These findings will contradict the findings in other studies about young children’s abilities to grasp and manipulate number. It does add to Mix’s conclusion that near the end of the child’s third year, the child becomes more able to engage in mathematical work that is symbolic and abstract and that occurs outside of social context.

Another interesting finding came out of Klein and Bisanz’s (2000) study. Some children spontaneously explained their reasoning although the experimenters took care not to elicit it. In approximately 30% of the trials in which children solved inversion problems using the covert strategy, the child explained that if you have to add a number and subtract the same number, then you really did not have to do anything. This finding suggests that four-year-old children are ready and eager to share the reasoning behind their mathematical strategies.

The purpose of Klein and Bisanz’s (2000) study was twofold. The researchers sought to understand what characteristics of math problems made them more or less difficult for children. The two-term problems helped the researchers answer the first question. The second question pertained to how children solved problems and whether or not they used shortcuts. Children’s solutions to three-term problems helped the
researchers glimpse the types of shortcuts that four-year-old children were capable of using to solve problems. Indeed, the findings revealed that children use a wide variety of strategies in solving problems requiring two procedures. Children used manipulatives, they used mental math, and they reasoned in other ways to solve these problems.

Klein and Bisanz’s (2000) study showed that, contrary to Piaget’s earlier findings (1952, 1971), four-year-old children do conserve numbers. They are capable of reasoning about numbers of items that are both out of their sight and out of meaningful social context. In fact, Mix (2002) found that Mix’s three-year-old son was capable of reasoning about number in meaningful social contexts. It could be that, while four-year-old children are capable of reasoning in abstract, arbitrary math problems, three-year-old children simply need problems to be situated in more meaningful context. If this is the case, it is credible that four year olds may demonstrate even more profound mathematical ability in socially meaningful contexts as well.

Mix (2002) found that the first understanding of numerosity that his son, Spencer, demonstrated in their home was that of one-to-one correspondence. Spencer initiated many one-to-one activities over the course of the second and third year of his life. Mix kept a diary of Spencer’s successes and challenges in practicing one-to-one correspondence. In fact, in the naturalistic setting of their home, Spencer was able to demonstrate understanding of one-to-one correspondence that he and other children his age were unable to demonstrate in clinical settings (Mix, 2002).

Understanding of one-to-one correspondence is a critical skill for learning to count (Mix, 2002). Piaget’s study of the children who could not quantify coins was more
or less a demonstration of the child’s failure to grasp one-to-one correspondence. However, Mix’s child spontaneously and successfully demonstrated one-to-one correspondence with graham crackers, silverware, steps, pours of sand, and raspberries on his feet—all before the age of three years old.

In the case of the silverware, Spencer handed out silverware, saying, “Mommy spoon, Daddy spoon, Mommy fork, Daddy fork” (Mix, 2002, p. 1349). He engaged in this one-to-one correspondence activity on countless occasions. Accomplishing one-to-one correspondence was slightly more difficult when passing out toys to children who were moving around at play. This finding recalls Mix’s 1996 and 1999 studies in which data revealed that children had more difficulty judging numerical equivalence sequentially than simultaneously because it was not possible to compare the two quantities side by side.

Many agree that children possess counting and other numeracy skills by elementary school age, but there is disagreement as to whether or not this ability is innate. Gelman (2000) argued that counting ability is innate, although it further develops as a result of experience. Without necessary experience, there is no way this ability can continue to develop. Others, like Piaget, insisted that counting and other mathematical ability was innate, although experience with the physical world did enable further cognitive development. It is interesting that Mix (2002) argued at length that infants and young children do not possess innate number sense. Rather, these children simply “bring the same basic sensory, motor, and learning capacities” to math problems that they “bring to any other task” (Mix, 2002, p. 1346). Mix thus disagreed with Gelman and
Piaget. However, Mix’s later study in 2002 called Mix’s own theoretical framework into question. In Mix’s 1996 and 1999 studies, Mix found that three-year-old children seemed to be unable to grasp number. However, in the 2002 study of Mix’s own son, Spencer demonstrated that he was able to grasp number well enough to understand one-to-one correspondence.

Mix (2002) pointed out that no previous study investigated the mechanics through which number concepts were eventually constructed. I have located research to contradict this contention. Cobb and colleagues (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996) investigated how number concepts continue to develop as teachers and children engage in conversation about numbers and operations, although these children were of kindergarten, first-grade, and second-grade age. Cobb did not examine how this process occurs with preschool-age children. Cobb and colleagues found that number sense developed as a social as well as psychological process. However, Mix’s studies (1996, 1999, 2002) are among the first in which number sense development was investigated in children under five years old. Mix focused on one-to-one correspondence activities and found that the child’s abilities were very dependent on the clues that social contexts bestow (graham crackers, toys, cups).

Spencer was 12 months old at the beginning and 38 months old by the end of Mix’s diary study. Mix (2002) recorded virtually all examples of Spencer’s use of one-to-one correspondence skill. The result was a rich and diverse set of examples. Mix, 2002 echoed Cobb’s finding that reasoning originated in social contexts: “... for
Spencer, the earliest one-to-one activities appeared to be embedded in social interaction” (p. 1351). It was interesting to note that only the last abilities to develop were those that other researchers had observed in laboratory settings. These laboratory studies involved aligning items with slots. Spencer performed poorly on such tasks until he was a full three years old. This poor performance might actually represent a validity issue with these tasks. In other words, these laboratory tasks may have failed to capture children’s one-to-one correspondence abilities. Before Spencer could engage in these tasks, he could tag objects, take turns, and tag events (Mix, 2002).

Mix (2002) noticed a limitation Spencer experienced in his attempts to label one-to-one correspondence between objects and people. When Spencer distributed toys among children, he could give each child a toy but he could not keep track of whether or not everyone had the same sized toy. Mix postulated that because the children were moving, their toys were not available to Spencer to closely inspect and pair with each child. Perhaps, as Piaget (1952) believed, children become confused by the global cues, such as the length of the line of coins. Here, Spencer was confused by the global cue of the number of children, and thus distributed trucks unfairly.

Mix (2002) noted that matching objects to slots may have been more difficult because the tasks were more arbitrary and out of social context. Even in a social context, Spencer used trial and error to distribute toys among children. He visited each child with a toy, sometimes even when he had already given one of them a toy. Then, seeing the person already had a toy, he moved on to the next person. This evidence suggests that Spencer required more practice in distributing toys equally. Moreover, he never
verbalized explicitly any facts of equivalence between sets. He only became agitated when a set was not equivalent, saying, “that’s Mommy’s!” if someone took away an object that was in front of her (Mix, 2002).

Mix (2002) delineated two requirements for this very young child to succeed in grasping one-to-one correspondence. First, the matches had to be “local” so that the child could focus on making one pair at a time. Two, the matches had to make sense in a social context. The social meaning of the action scaffolded (Wood, Bruner, & Ross, 1976) the child in making the match. At the finish of her study, Mix (2002) concluded that children did not slowly develop competence in one-to-one correspondence. Rather, they demonstrated complete competence within a narrow range of activities. As time went on, children made mental connections between these contexts in order to grasp number more abstractly. Children’s concept of number is not inferior or incomplete. It simply appears in places researchers may not have previously expected. These findings suggested that children older than three years old may be better prepared for school mathematics. However, appropriate contextualized activities may also be prepared for children under the age of four.

It was also important to note that Spencer spoke spontaneously about the real life math problems he was solving. However, Mix (2002) once again failed to elicit or explicitly study this aspect of Spencer’s experience. Once again, Spencer often spoke spontaneously about what he was doing. Another study might have drawn out this child’s natural and learned thinking about how to solve problems of one-to-one correspondence. This naturalistic diary of Spencer’s solution of problems in his own
home was the most relevant cognitive science study for the current one, which also took place in a naturalistic setting. Opportunities that were lost in this study to explore Spencer’s thinking were closely examined through observations as well as interviews in this study.

Barth and colleagues (2005) conducted a more recent study on cognition and number sense. These researchers found that children possessed robust ability to compare and add large quantities. Furthermore, these preschool-age children were capable of judging quantity across modalities (objects and tones). The participating children were successful in comparing large quantities, even when the experimenters controlled for continuous variables such as area or line length. The study consisted of five experiments that involved four- and five-year-old children. Each experiment involved 16 to 17 children. In each, the children were told they were to play a computer game.

In their first of the five experiments in this study, Barth and colleagues (2005) sought to observe whether five-year-old children could compare large sets of dots based on numerosity and not based on continuous variables. They also wanted to observe whether or not the accuracy of children’s comparisons depended upon the ratio between the sets. In the first experiment, on a computer screen, children were shown large quantities of red and blue dots. The arrays included 10 to 58 dots. Red dots were shown first and then blue. The children were told to judge whether there were more red or blue dots. As anticipated, the children did experience slightly more difficulty in comparing sets accurately when the ratio between the sets approached one. However, they succeeded significantly above chance as long as the ratio between the sets was 3:4 or
smaller. However, the children answered correctly well above chance (Barth et al., 2005).

As in previous studies (Klein & Bisanz, 2000; Mix, 1999; Mix et al., 1996), children were looking at arrays of objects that were quite arbitrary and out of socially meaningful context. However, they performed even more successfully on these tasks than did the participants in Klein and Bisanz’s (2000) study, which also included sequential presentation of dots. This sequential presentation made the task difficult. It is much easier for children to compare quantities when they are side by side (Mix, 1999, 2002). Also, the dots were shown too quickly for the children to count them. The children were thus forced to rely on their innate ability to approximate number.

In experiment one, and in all the subsequent four experiments in Barth et al.’s (2005) study, children were not statistically confounded by continuous variables such as area of the screen covered by the dots or by the contour of the line created by the array of dots. For example, when the red dots were spread out across the screen, this did not confuse the children as Piaget (1951, 1952) would have predicted if there were less red dots than blue dots. The preschool-age children involved in this study were statistically successful in all five experiments in the study.

In experiment two, children were sequentially shown two arrangements of dots. They were told to add these first two quantities and then compare the sum to a third array. Some children were biased in order to judge the sum of the first two as being more numerous. However, statistically, the children chose the correct quantity as being larger (Barth et al., 2005).
In the third experiment, the children were shown an array of dots. Then they were presented with a quantity of tones. They were asked to compare the numerosity of the dots with that of the tones. The preschool-age children performed successfully on this cross-modal comparison. It was thought that perhaps formal experience with arithmetic and symbolic mathematics would be necessary for children to understand that the concept of number united both objects and tones. However, this experiment demonstrated that preschool children possess conceptual understanding of number even before they are formally introduced to operations (Barth et al., 2005).

The fourth experiment built upon the first three by calling on children to add two sequentially presented arrays of dots and compare the sum with a number of tones. As with the other three tasks, the children performed reliably above chance. This finding proved that preschool-age children were capable of engaging in abstract addition using large numbers. Other studies have shown that preschool-age children can compare, add, and subtract even large numbers of items in visual arrays (McCrink & Wynn, 2004). However, Barth and colleagues (2005) demonstrated that children could add and compare quantities across modalities.

In experiment five, the experimenters simply presented the children with word problems containing symbolic numbers. They did so in order to ascertain whether or not the children had somehow learned how to engage in symbolic mathematics without having learned it in school. In one example, the experimenter asked the child, “If your mom gave you twenty-seven marshmallows, and then she gave you thirty-one more, how many would you have?” (Barth et al., 2005, p. 14120). The experimenter offered the
child options: “Would it be more like fifty-eight or thirty-three?” (Barth et al., 2005, p. 14120). Children did not answer these questions correctly above chance. This finding suggested that children’s correct answers in the previous experiments were the result of an ability to approximate quantity and not a result of formal arithmetic. However, as Dehaene found, more formal, culturally-specific understanding of mathematics could be built upon the foundation of this core comprehension of number.

Barth and colleagues (2005) argued that their findings, along with others’ findings (Klein & Bisanz, 2000; Mix, 1999, 2002) disproved Piaget’s contention that children are incapable of conserving number until the age of seven. I would go further and argue that if children are indeed capable of conserving numbers, then it is logical that they can capably converse about the process by which they have conserved number. This study and the others detailed in this section suggest that children possess sophisticated premathematical approximation skills. These skills could conceivably enable children to engage in substantive discussion about the reasoning they used to solve mathematical problems.

It is important to add an important caveat to the previous discussion: researchers have found considerable variation in the number skills of children of preschool age (Klibanoff et al., 2006). Some demonstrate an array of number skills, counting elements in sets, matching sets in terms of cardinality, order sets by numerosity, and carrying out rudimentary calculations. Statistically, children from middle- and high-socioeconomic status (SES) demonstrate higher levels of mathematical abilities than lower SES children of the same age (Jordan, Huttenlocher, & Levine, 1992). These differences of
mathematical ability are compounded by linguistic differences (Moschkovich, 2010; van Kleeck, 2014). Because lower-SES children arrive at school with less-developed, school-appropriate language abilities, they are less able to access mathematical conversations that are necessary for the further development of mathematical reasoning.

Early differences lead to continuous implications for school achievement in later years (Denton & West, 2002). These differences last into middle as well as high school (Braswell et al., 2001). The subsequent sections of this chapter describe some of the mechanisms underlying this trajectory of underachievement over the course of the early school years. Low early math and language math achievement leads to exclusion from mathematical conversations (Barletta, 2008). Exclusion from mathematical conversation can lead to disenfranchisement in the classroom microcommunity. Reasoning develops as children internalize classroom norms for mathematical reasoning (Yackel & Cobb, 1996). It follows logically that disenfranchisement in classroom communities may lead to lost opportunities to internalize ways of reasoning. Fortunately, as detailed in the final section of this chapter, there are conversational strategies that teachers can use to include more student voices in classroom communities and hopefully curtail the continuation of this vicious cycle (Chapin & O’Connor, 2007; Temple & Doerr, 2012).

Academic Language and Children in the Linguistic Minority

All students must master the language of the classroom in order to be academically successful (Cazden, 2001; Francis, Rivera, Lesaux, Kieffer, & Rivera, 2006; Gee, 1996, 2014a, 2014b). Some children come to school already conversant in mathematical language. In the classroom, these students are in the linguistic majority
(Barletta, 2008; Heller, 2006; van Kleeck, 2014). These children have been exposed to academic language in the home and thus speak it at school. These children easily join in communication with the teacher because they speak similarly to him or her. The teacher expects children to speak in academically appropriate ways, even if they are unaware of this expectation.

Other children who come to school lacking familiarity with school-appropriate talk are considered by some researchers to be in the linguistic minority (O’Neal & Ringler, 2010). An alarming finding is that students who need the most practice with academic language often receive the least (Lagrande & Reid, 2000). These linguistic minority children are shortchanged in the classroom. Their voices are not heard, and thus they are unable to join in authorship of the classroom culture (Yackel & Cobb, 1996). The result is a classroom culture that does not reflect their ways of thinking and speaking mathematically. A vicious cycle ensues in which the classroom culture excludes them further and further as time goes on.

The experiences of these children can be explored through the lens of Vygotsky’s (1934/1978) theory. All students learn optimally within their zone of proximal development (ZPD). At the bottom of the ZPD, children are capable of solving problems independently. At the top of the ZPD, children are only able to solve problems with the help of a knowledgeable other. According to Vygotsky, students must be taught at the top of their ZPD in order for them to move forward intellectually. In the case of linguistic majority students, teachers communicate frequently with them, scaffolding their math development at the top of their ZPDs. In the case of linguistic minority
children, teachers fail to engage them. Thus, these students are not being taught at the
top of their ZPDs.

Parents’ Use of Abstract Language at Home
van Kleeck, Gillam, Hamilton, and McGrath (1997) had previously established
that middle-class parents’ abstract language use plays a vital role in children’s
appropriation of abstract language development. Fluency in abstract or decontextualized
language was critical for children to later engage in academic conversations at
school. This study, like some of those cited above, confirmed that experiences prior to
schooling may determine children’s abilities to engage in academic language. In this
study, van Kleeck and colleagues videotaped parents and children in their homes. These
participating parents read both familiar and unfamiliar books to their preschool-age
children. As they read, these parents discussed the text with their child. These parental
discussions of the text were coded for level of abstraction. Next, these levels of
abstraction in parent speech were correlated with the gains of children one year
later. Findings showed that the level of abstraction parents used in these discussions
were significantly correlated with children’s scores at the end of the experiment. The
children of parents who used more abstract language scored better on abstract language
measures.

Van Kleeck and colleagues (1997) gave 13 girls and 22 boys a pre- and posttest
called the Preschool Language Assessment Instrument (PLAI). Subsequently, parents
read books to their children and discussed their contents. The book-relevant discussion
was coded for naming talk, literal descriptions, and inferential talk. Other researchers
have looked at parent-child book-sharing (Scarborough & Dobrich, 1994). However, not all examined the levels of abstraction in parents’ speech. Further, only two of these studies related abstraction of parent speech to children’s subsequent development of abstract language (Heath, 1983; Sigel & McGillicuddy-DeLisi, 1984). In this and other studies, van Kleeck and colleagues (1997, 2011, 2014) found that early experiences at home lead to children’s different levels of comfort with school-appropriate talk.

The pioneering work in the area of class and abstract language development was a longitudinal ethnographic study by Heath (1983). This researcher compared two middle-class and one lower-class communities in terms of how well parents prepared their children for the demands of school. Heath (1983) found that only the middle-class community adequately prepared their children for school. While each community socialized children into language practices that were functional in their home community, children from the two low-income communities struggled once they were in school. Race was not a major determining factor according to these findings. One of the low-income communities was African American and the other European American. Children from both communities struggled in school. Heath (1983) posited that middle-class children learned sophisticated ways of interacting with texts in interactions with their mothers. Parents “in town” read books aloud to their children, but also went further by asking questions and making predictions about stories. By thinking aloud about the text, these middle-class parents led their young children to think more abstractly about texts even before entering school.
Book-sharing time is one very important activity for socializing children into the use of abstract language. However, other researchers have examined parental modeling of abstract language during other activities, such as during mealtimes (Beals, 1993) or during play with play-doh (Sorsby & Martlew, 1991). van Kleeck and colleagues (1997) suggested that book-sharing is the most likely time that parents might scaffold (Wood et al., 1976) children’s development of abstract language because there was no motor activity to distract them. The research of van Kleeck and colleagues (1997, 2011, 2014) suggested that middle-class children enjoyed innumerable opportunities to develop abstract language before arriving at school, perhaps putting lower-income peers at a disadvantage.

Academic Language and Literacy

Some of the research on academic language has investigated how education professionals help the youngest children (preschoolers and kindergarteners) to learn this language in the context of literacy lessons (Cain & Oakhill, 2007; Cain, Oakhill, Barnes, & Bryant, 2001; Cain, Oakhill, & Elbro, 2003; van Kleeck, 2008). Many of these studies looked at how educators supported the development of academic language through teaching inferencing skills. While many of these studies suggested implications for mathematical language development, none of them addressed this development directly. There is a gap in the literature that must be filled in order to understand how academic language functions in the mathematics classroom. Although the work of van Kleeck (2011, 2014) focused on reading and book sharing, the ultimate focus was on school talk and reasoning, and thus there are clear implications for mathematical talk and reasoning.
Layers of Linguistic Difference

Most research on linguistic differences in the classroom pertains to students who struggle because they are still learning the English language (DiCerbo, Anstrom, Baker, & Rivera, 2014; Moschkovich, 2007). Other researchers examined students who spoke dialects of English other than Standard American English (Edwards et al., 2014; Labov, 1972) and how this linguistic difference could pose a challenge for learning academic language. For example, speakers of African American English might find themselves left out of academic conversations because of their linguistic minority status.

However, the concept of school talk adds a more nuanced layer to the problem of linguistic difference in the classroom. A student can be an English speaker, and even a speaker of Standard American English, and still lack fluency in the specific vocabulary, grammar, and other language conventions associated with mathematical (Halliday, 1978; Pimm, 1987). Because knowledge of this specific style of speech is necessary in order to comprehend and communicate academically (Cazden, 2001; Gee, 2009), teachers must teach it explicitly (Chapin & O’Connor, 2007) or teach in such a way that makes the workings of academic language explicit (Temple & Doerr, 2012). The following sections describe some of the characteristics of academic language.

Features of Academic Language at the Preschool Level

Very little research exists on educators’ use of mathematical language with preschoolers (Klibanoff et al., 2006). However, research on academic language use with preschoolers is growing (van Kleeck, 2014). It should be noted that much overlap exists between casual or everyday talk and academic talk. Other vocabulary words,
grammatical structures (Schleppegrell, 2004), as well as social-interactive and cognitive conventions (van Kleeck, 2014) are specific to academic talk. Here, I will outline the features researchers have identified as being particular to the academic language that teachers use with preschoolers. See Figure 1 for an illustration of how teacher conversation strategies assist students in the linguistic minority in improving their academic language skills within their ZPDs. This assistance occurs in the context of the classroom microculture.

![Figure 1. Theoretical framework of current research.](image)

It is important to note that the features particular to academic language are also often present in the home language of White, middle- and upper-class students. Thus, academic language favors these students more than students from other demographics. In a synthesis of literature on preschool classrooms, van Kleeck (2014) described features of academic language, which are related in the following sections. Halliday (1978) and
Pimm (1987) described mathematics language in strictly linguistic terms. However, more research is required to describe the characteristics of mathematics language in terms that benefit early childhood teachers who do not have a background in linguistics.

The language of school is quite similar to the language spoken in White, middle-to upper-middle class homes (Barletta, 2008; Lagrander & Reid, 2000; van Kleeck, 2011). Thus, these students more easily join in on academic conversations. Teachers, for their part, rewarded and reinforce these students’ efforts. Conversely, they tend to dismiss student contributions when those students speaking are linguistic minorities (Heller, 2006).

In order to scaffold (Wood et al., 1976) the development of students’ academic language, it is important to review some of the special features of this language. The features outlined below describe academic language. Many of them describe more specific mathematical language as well (Halliday, 1978; Pimm, 1987).

Autonomy. Research suggests that U.S. teachers often expect even the youngest students to be autonomous thinkers and speakers (Degotardi & Torr, 2007). Students are expected to express novel opinions as opposed to simply repeating what has already been said. These school cultures value Western individualism over Eastern collectivism. Children are expected to possess unique interests and opinions and to verbalize them.

Williams (1999, 2001) examined opening clauses of teacher speech and found that these teachers often modeled verbalization of states of consciousness, saying, “I wonder if . . .” or “I hope that . . . .” In research summarized by Barletta (2008), teachers positively reinforced the speech of students whose speech resembled their own and often
dismissed students whose language did not. Thus, students from cultures that value collectivism more than individualism may have been at a disadvantage. Some refer to these marginalized students as *linguistic minorities* (O’Neal & Ringler, 2010). In this sense, and in others described in the following sections, students can become linguistic minorities in their classrooms.

Verbal display. Another expectation that preschool teachers have demonstrated is for students to display their knowledge verbally (Scollon & Scollon, 1981). In some cultures, young children are expected to passively listen to those who possess more expertise than themselves. However, in U.S. schools, teachers deliberately ask questions they already know the answer to in order to elicit answers from young children. The purpose of this practice is for the asker to discover how much or how little the child has learned. Thus, those who supposedly know the least are often in the role of the exhibitionist rather than the spectator. Students from cultures in which young children are accustomed to being spectators may be at a disadvantage.

Heath (1983) found that working-class, African-American children were not socialized into this practice. In fact, when adults asked questions children already knew the answer to, it was usually to chide them. However, verbal display was present in music like hip-hop and rap. The function in these forms of music are not to provide information to an authority figure. Unfortunately, Heath (1983) noted, these amazing performances were rarely examined or praised in the classroom setting. Heath also noted that known-answer questions were mostly absent from the socialization of Mexican-American children. These adults only asked known-answer questions to tease children.
Teachers may also expect preschoolers to verbally display not just their knowledge but also their thought processes. van Kleeck (2006) found that parents sharing books with preschoolers modeled discussion on known information 60% of the time and on making inferences 40% of the time. These two foci reflect the two goals of helping children feel successful and also building upon prior knowledge (van Kleeck et al., 1997). Researchers suggest that these two types of linguistic interaction be used judiciously and in balance (Cazden, 2001). Teachers often expect children to verbally display such thought processes while reading.

Formality. It may go without saying that academic language is more formal than is casual language (van Kleeck, 2014). It is said to be more impersonal and more objective. The vocabulary of academic language is more literary and Latinate. Casual language draws from more Germanic lexicon (Bar-Ilan & Berman, 2007). Words and phrases like *smart, land, huge, kid, and get* are common in casual language. Words like *relinquish, fidelity, sequence, inquire, and cognizant* are likely to be found only in academic language.

Further, contractions and personal pronouns are more likely to be found in casual language, especially the first person nominative, I. Casual language contains more appreciative markers that communicate personal interest (Gee, 2005). A variety of moods are found in casual language (declarative, interrogative, imperative), giving it a more informal sound. The declarative is the most common mood found in academic language (Schleppegrell, 2001). The active voice is most common in casual language, while the passive voice is most common in academic language (Biber, 2003).
Discreteness of objects. Once again drawing a distinction between Western (individualistic) and Eastern (collectivist) culture, Nisbett (2004) described cultural differences between these two worlds by noting that in the West, thinkers value linear logic. They conceive of the world through analysis of cause and effect relationships. In the West, thinkers compartmentalize objects, people, and phenomena so that discrete items can be described in terms of their proper attributes and then categorized separately and neatly.

Cause and effect relationships can then be described as occurring between these discrete objects. This way of conceiving of the world lends itself to quantitative study. This study, being qualitative, nevertheless ran up against this way of seeing objects, people, and phenomena. These were conceived of as being separated instead of entirely interconnected as they are more likely to be seen in the East. Teachers and researchers who are members of Western culture are likely to view students and their objects of reflection in this divergent light. Academic language thus reflects the Western view of phenomena (van Kleeck, 2014), whereas casual language may not reflect this view as sharply.

Degree of contextual support. The distinction described in the section Discreteness of Objects is related to the question of context in speech. Speakers of academic language tend to rely less on contextual support, whether from physical or social context. Casual talk usually describes more of the immediate physical or social context. Objects of discussion are thus taken as inseparable from their context. Listeners rely on this context to understand the speaker’s meaning.
These last few sentences that I have written were written in academic language to the extent that they described the topic of discussion in abstract or decontextualized terms, as opposed to describing actual places or actual speakers, with examples of their speech. Even in preschool, teachers begin to rely less in their speech upon specific contextual information (De Temple & Beals, 1991; McKeown & Beck, 2003; Watson, 2001). Language used in school becomes increasingly decontextualized over the course of students’ elementary, high school, and college careers. Decontextualized speech places a greater demand on both speakers and listeners as it forces these participants to create mental representations of objects of conversation.

Degree of generality. Simultaneous with the abstraction of speech, academic language also refers to a more general level of thought. Casual speech usually pertains to specific people, places, and things because the goal of this speech is to accomplish daily goals. Academic language, by contrast, categorizes these objects and then describes them in terms of more general characteristics.

This mode of speech enables speakers and listeners to form scientific theories about these categories of objects (Bruner, 1966; Westby, 1985). The different aims of these forms of speech result in the use of different lexicon as well as different grammatical structure (Schleppegrell, 2004; van Kleeck, 2014). However, the formal linguistic differences of speech associated with academic language and casual language are not the subject of this study.

Degree of precision. Because language becomes more precise in academic language, speakers use less of the ambiguous terminology used in casual language, such
as sort of or kind of (Snow & Uccelli, 2009). Students as young as preschoolers begin learning tier three words such as photosynthesis, eclipse, evaporation, or metamorphosis. In mathematics lessons, they may be exposed to words like addition, fraction, or half. These words are strictly domain specific. They are also morphologically complex compared to words used in casual language (Nippold, 2007).

Differences in reasoning style. In casual language, statements of belief do not always require justification (Reif & Larkin, 1991). Furthermore, the speaker is not required to address alternative points of view. Speakers may often ignore and choose not to bring up evidence that contradicts their views (Kuhn, 1991). Academic language, on the other hand, relies on formal organization (Reif & Larkin, 1991).

Speakers of academic language offer evidence for their positions, addressing contradictory information in order to refute it formally (Kuhn, 1991). Casual speakers are not typically expected to be totally confident in the views they are expressing, while academic speakers are expected to have a high degree of confidence in their view before speaking (Snow & Uccelli, 2009).

Casual language tends to be characterized by topic association, whereas academic language is characterized by topic focus. Casual speakers are expected by their audiences to tell a good story. This can result in narratives that move from one topic to another associated topic in a kind of chain of related ideas (Bruner, 1966). Academic speakers, by contrast, are expected to form an organized argument to convince their audience of some belief or idea. Thus, their narratives are linear and focused, delving deeply into a single topic.
Research has investigated preschool children’s narrative styles in the context of show and tell (Barletta, 2008; Michaels, 1983). African American students often narrate in a topic-associative style during show and tell (Hyon & Sulzby, 1994). These students’ teachers may be unaware of the different forms that narratives can take. Thus, they struggle in helping these students to reorganize their narratives to make them topic-focused (Barletta, 2008; Cazden, 2001). Because of their lack of information, teachers may mistakenly evaluate these students as being incapable of organizing narrative. In the worst-case scenario, teachers may interrupt the child’s narrative multiple times so that their narrative loses any comprehensible organization. Barletta (2008) referred to this process as the dismantling of narrative.

The organization of narrative has special relevance for mathematical explanations. Both narrative and mathematical explanations offer listeners an explanation of cause and effect. In fact, mathematical explanation is a form of topic-focused narrative. In offering a mathematical explanation, the speaker is offering listeners a well-formed argument in order to convince them of some mathematical truth. Therefore, children whose narratives are topic-associative (Barletta, 2008) may be at a disadvantage in math class. Conversely, students whose narratives are topic-focused will be at an advantage. According to Barletta (2008), teachers will call upon students whose language is already appropriate for school more often. Students whose language the teacher deems inappropriate will be dismissed and called on to speak less. The result of this prejudicial treatment is that students who need the most practice speaking in school-appropriate ways receive the least, while those who are already capable of
speaking in school-appropriate ways receive more, gaining ever greater competence Barletta (2008).

The occurrence of this unjust treatment (Barletta, 2008; Michaels, 1983) is the reason that one research question in my study pertains to individual linguistic differences and the results for each student’s success in joining in mathematical conversations. In observing a preschool teacher’s successes and challenges in orchestrating classroom conversation, it is important to consider ways this teacher supports linguistic minority children in accessing mathematical conversations.

In observations, I paid attention to times when children were successful in speaking academic language and times when they are not successful. Observations and interviews with the teacher focused on ways she scaffolded these students in appropriate ways to help each student to be more successful in reasoning and speaking mathematically. Linguistic differences were mainly a result of different home languages and not a result of SES. All the children in the class were of high-SES background. However, in the spirit of ethnographic research (Spradley, 1979, 1980; Wolcott, 2008, 2010), inquiry remained open to many possible sources of linguistic difference.

Level of reasoning. Teachers of preschool generally require lower-level literal reasoning and speech as opposed to higher-level inferential reasoning and speech (van Kleeck, 2008; van Kleeck et al., 2006). However, it is important to note that language at school does involve more higher level, inferential reasoning than does reasoning outside of school (Reif & Larkin, 1991). Preschool is no exception.
Literal language is relatively easy for children to master. In literal language, children may be asked to talk about a person, place, or thing directly in front of them. However, inferential language requires that children fill in missing information or connect the dots in a way that is not made immediately apparent. This type of language requires that children problem solve, hypothesize, compare and contrast, and engage in many other challenging mental tasks (van Kleeck et al., 1997).

The Importance of Teaching Academic Language

Children must gain fluency in academic language in order to be academically successful (Cazden, 2001; Gee, 2006). Differential home experiences mean that children come to school with different language abilities. Researchers have found that student linguistic differences were associated with race and culture (Barletta, 2008; Heath, 1983; van Kleeck et al., 2011) and even more important differences because of class (Heath, 1983; van Kleeck, 1997). Research has shown that teachers were more accepting of the speech of White, middle- and upper-class children (Barletta, 2008). As a result of this documented prejudice, students who needed the most practice with academic language often received the least. Teachers may not have known where to begin in scaffolding (Wood et al., 1976) students’ development of academic language. Recent studies offered examples of discourse-rich mathematics instruction from middle- and high-school classrooms (Chapin & O’Connor, 2007; Temple & Doerr, 2012). More investigation is needed into how preschool teachers might engage students in productive mathematical discourse at the preschool level. This study began to meet this need. The following
section offers a discussion of recent investigations into the conversational strategies that teachers have used during mathematics lessons.

**Teacher Conversation Strategies**

Children must learn academic language in order to be successful in school (Cazden; 2001; Gee, 2009), including in mathematics. However, many children have failed to develop the academic language necessary for them to achieve success in mathematics. A number of factors have contributed to this grave social problem. Some children do not speak English at all, but still others speak a nonstandard dialect of English (Labov, 1972). Other children speak Standard American English, but do not have as much practice at home in speaking academic language as some other children did. For their part, teachers at the early childhood level may not be skilled in engaging children in academic conversations (Barletta, 2008). Research has shown that teachers should engage children in conversation during mathematics class in order to facilitate development of mathematical reasoning (Walshaw & Anthony, 2008). Through the intentional use of conversational strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012; Wood, 1989), teachers have been able to extend talks and include all children in focused mathematical dialogue. In this way, teachers have been able to ensure that all students gain the practice they need in verbal mathematical reasoning.

Teacher-centered discourse continues to be the norm in elementary and high school classrooms in spite of research recommending student-centered discourse and instruction (Cuban, 1993). In other words, teachers still do most of the talking in the classroom. Students should do more of the talking (Yackel & Cobb, 1996) in order for
real conversation to take place. Authentic conversation ensures that children internalize conceptual knowledge of mathematical ideas (Vygotsky, 1978). Because teachers lack skill in engaging students in genuine conversation, most classroom conversation follows the Initiation-Response-Evaluation (IRE) format (Mehan, 1979; Sinclair & Coulthard, 1975), with teachers asking low-level questions, prompting students to recite facts and rules or follow rote procedures (Hiebert et al., 2003). Teachers initiate interactions, students respond, and the teacher ends the interaction by evaluating whether the student is right or wrong. This type of interaction is superficial and inauthentic. Students have not been called upon to press for true conceptual understanding, which can only occur in extended conversation (Yackel & Cobb, 1996). Instruction remains a mile wide and an inch deep. Perhaps teachers have had trouble breaking out of this traditional format because they were never exposed to different models of classroom conversation.

Some research exists on the practices teachers use in order to engage children in mathematical conversation. Researchers have not agreed on what successful teacher conversation strategies sound like. The following sections detail several models. It should be noted that because teacher conversation strategies is a relatively new area of research, the data presented is qualitative. More quantitative data is necessary to understand the relationship of various strategies to student outcomes. It could be that all of the described models for classroom conversation provide useful information for teachers seeking to enrich mathematical discourse in their classrooms. In many of the offered examples, conversational strategies were used to successfully extend classroom conversations and to include more students in these conversations.
Some research has addressed teacher questions (Franke et al., 2009; Wood, 1989). One researcher took a broad view of teacher questioning sequences, dividing them into two broad types: probing or leading (Franke et al., 2009). Another examined the same dichotomy, labeling the speech types *funneling* or *focusing* instead (Wood, 1989). This research on question sequences represented the simplest way of looking at teacher conversation strategies. In these studies, the researchers investigated how teachers used questions as tools to move students toward complete and coherent mathematical explanations. Other researchers divided teacher conversation strategies into five types, or tools, which they called *talk moves* (Chapin & O’Connor, 2007). Finally, Temple and Doerr (2012) delineated at least 12 types of interaction types, offering transcripts to illustrate each.

Teacher Questions

In the classrooms studied by Franke and colleagues (2009), the teachers all asked their students to share their thinking about problems during whole class discussions. These teachers pressed students to furnish explanations for their strategies. The researchers found that the type of questions the teachers asked usually led to very different teacher-student interactions. It was ultimately found that students were most successful when their teachers struck a balance in the types of questions they asked during mathematical conversations. Franke and colleagues (2009) described four types of question sequences: probing, general, specific, and leading. Each type of questioning sequence led to different teacher-student interactions. Two types, probing and leading questions, can be clearly contrasted. These are the two I will examine here.
Franke and colleagues (2009) studied three classrooms. They analyzed the questions that teachers to help students explicitly state and elaborate upon their mathematical thinking. Two were second-grade classrooms and one was a third-grade classroom. All three classrooms were situated in different elementary schools in a large school district in Southern California. This district suffered from a history of low student performance. It was indeed one of the lower performing districts in the state. However, the three teachers chosen were notable because of the productive conversations they succeeded in orchestrating in their mathematics lessons. These teachers used question sequences in different, beneficial ways. The findings of this qualitative study show that different types of question sequences prove effective during mathematics lessons. They also show that productive mathematical conversation can make a difference in schools with struggling students.

These three teachers taught in similar schools, shared similar pedagogical agendas, and structured their classrooms similarly (employing both group and whole-class discussion to analyze problem solution). However, the scores of these students on algebraic thinking varied substantially. Franke et al. (2009) used both video and audiotape to analyze differences in the teachers’ discourse practices to possibly account for these differences. Four types of question sequences were found to lead to different kinds of student participation. The two that can be sharply contrasted were probing and leading questions.

Probing questions were open-ended, while leading questions were far more closed-ended. In probing questions, teachers uncovered student errors in problem-
solving strategies, highlighted a student’s use of a correct strategy, or clarified a correct strategy that may have been slightly off the mark. Teachers often used a probing sequence of questions when a student’s explanation had some potential but was initially unclear. These sequences of questions prompted students to elaborate further and often afforded them the opportunity to offer correct explanations of their own problem solving strategies (Franke et al., 2009).

In this example, the teacher posed the problem: \(200 + 1 = 200 + 1\) or \(200 + 1 = 1 + 200\). Does it matter that these numbers are in different order? The student, Krystal, answered that the order did not matter because the numbers all have “partners”. This resourceful phrasing was met with approval by the teacher. However, the teacher pressed the student to explain more precisely what she meant. The teacher, Ms. Guo, asked Krystal, “What are you talking about? Could you explain what numbers you are talking about?” Krystal then explained more clearly, “The one and the one are partners and the two hundred are partners” (Franke et al., 2009, p. 386)

The teachers asked more specific questions or leading questions when there was a specific aspect of a strategy that they wanted the students to attend to. Unlike in the probing question sequences, the student was not ultimately responsible for offering the complete explanation. Instead, the teacher took most of the responsibility for presenting an explanation, asking the student to focus only on one aspect of it at a time. This approach ensured that the student learned a specific aspect of the problem that was on the teacher’s agenda. In one such sequence, the teacher, Ms. Guo, posed this problem: \(100 = 50 + \_\). The student, Lauren, explained that she thought the answer should be
fifty. Ms. Guo talked her through an explanation of this correct answer by saying, “Fifty plus fifty equals one hundred and so one hundred is the same as. . . .” Lauren answered, “One hundred” (Franke et al., 2009, p. 389.)

Five Talk Moves

Not all teachers are equally effective in orchestrating classroom dialogue (Chapin & O’Connor, 2004, 2007; Lopez & Allal, 2007). Chapin and O’Connor (2007) identified ways teachers effectively orchestrated mathematical conversations during a four-year, federally-funded program entitled Project Challenge (Chapin & O’Connor, 2004, 2007; Chapin, O’Connor, & Anderson, 2003). Chapin and O’Connor (2007) described the practices of one effective teacher, detailing the five talk moves, or conversational strategies that she and her students employ during productive mathematical conversation. The teacher made use of these talk moves to build upon students’ verbal contributions, either to showcase these contributions, refine them, or offer them up for rebuttal. Chapin and O’Connor (2007) found that successful teachers made use of these talk moves, in addition to establishing norms of respect and inclusion. In the context of these respectful communities, students developed understanding of mathematical concepts and procedures.

The U.S. Department of Education funded Project Challenge under the Jacob K. Javits Act. This project was undertaken in order to help at-risk elementary and middle-school students. The intervention took place between 1998 and 2002 in a low-income urban school district located in the Northeast. The annual project enrolled 100 students in grades four through seven. The majority of the students was English language
learners, and most qualified for free- or reduced-priced lunches. Over the course of this funded project, Chapin and O’Connor (2007) worked with 18 teachers and about 400 students. Teachers in these classrooms maintained a focus on student explanations for their mathematical reasoning.

At the end of each year of the project, the experimenters administered the California Achievement test (CAT), a nationally normed standardized test. The students who took this test demonstrated gains in achievement year after year (Chapin & O’Connor, 2004, 2007; Chapin et al., 2003). This project furnished both quantitative and qualitative data demonstrating the importance of mathematical talk. The intervention was not initially set up as a randomized controlled experiment. However, the researchers later conducted a post hoc quasicontrolled comparison. This study demonstrated that students not enrolled in the program performed on a significantly lower level on the CAT as well as on the yearly state assessment (Chapin & O’Connor, 2004). In my study, the teacher employed the same talk moves as the teachers in Chapin and O’Connor’s study, but there was no pretest or posttest. More research is necessary to determine whether or not these talk moves would help to improve preschoolers’ achievement scores.

There were many components to the program created by the experimenters: the use of a constructivist mathematics curriculum, professional development experiences for participating teachers, and what the experimenters considered the most powerful component of the program: identification of specific communication practices used by the teachers. Not all talk is productive in and of itself. The kind of talk typified by the conversation that follows was academically productive. It was productive in the sense
that it supported students’ development to both reason and to put their reasoning into words. The researchers found that productive mathematical talk was typified in one teacher’s sixth grade class. The experimenters considered this teacher, Mrs. Holden, to be an expert in the use of classroom discourse. Mrs. Holden and her students made use of the following talk moves in order to move mathematical conversations forward:

(a) “Revoicing” is a strategy that can be used by both teachers and children. To use this move, the speaker restates an utterance of a previous speaker, usually to make it more clear. This move is used effectively when the speaker asks the previous speaker if the restatement is correct (e.g., “I think you are saying...is that correct?”) (O’Connor & Michaels, 1993, 1996).

(b) Teacher requests for a child to repeat another child’s contribution. (Connie, can you repeat the statement that John just made?)

(c) Teacher-initiated elicitation of children’s reasoning (e.g. What do you think about what Connor just said? Why do you disagree?)

(d) In the “adding on” talk move, the teacher requests for a child to elaborate or add on to what another child has said. (Does anyone have something else to add to what Hannah’s explanation?)

(e) Teacher wait time. Effective teachers often leave lengthy wait times after calling on some students. A few seconds may not be long enough for some students to formulate meaningful contributions. Some students need longer to contribute, but the result will be more inclusive conversation.
At the beginning of an extended conversation, Mrs. Holden wrote a number on the board: 215. She asked the students if the number was odd or even. She also asked them to explain how they arrived at their answer. Mrs. Holden called on a student named Kimberly, who stated that the number was even. Kimberly’s answer was brief. She did not use precise language. She noted simply that it was even “because there’s a two” (Chapin & O’Connor, 2007, p.116). Next, the teacher called on Dan, who spoke in slightly more precise terms. Dan stated that the number was odd, because when divided in two, there would be a remainder of one. Dan’s answer was correct. However, Mrs. Holden made a point of extending the conversation, seeking to include more student voices in the conversation. She called on another student, Brian, to ask if he agreed or disagreed with Dan’s answer. Brian agreed, but added that he knew the number was odd because the last number was odd.

Mrs. Holden asked a student named Rita what she thought. Rita added even more information to the answer. She stated that the digit in the ones place made the difference. However, she added a slightly confusing comment, saying that the first number was the important one. Mrs. Holden pinpointed this comment as an important one to discuss. “I’m a little confused,” Mrs. Holden said (Chapin & O’Connor, 2007, p. 116). She posed the question to the class, is it the first or the last number that is important in determining whether it is an even or odd number?

In pulling out this point, Mrs. Holden was potentially identifying a point of confusion for many of the students. After all, two of the students she had called on had commented that the first number mattered. At this point in the conversation, each student
had added a little bit of information to the discussion. Even though she was looking at the wrong digit, Kimberly knew that looking at a single digit in the number could help determine whether or not the number was odd or even. Dan added the idea of dividing the number in two. Brian clarified that it was important to look at the last digit. Finally, Rita added the use of the word, “digit”. Up until that point, the students were not referring to digits with that precise mathematical term. So far, Mrs. Holden was successful in engaging a number of students in conversation. The students, for their part, were successful in adding nuances and shades of difference to the discussion of how to discern even and odd numbers. Even student misconceptions were used to help the whole class clarify and build knowledge about numbers.

The conversation continued, with more students contributing. Although one of Mrs. Holden’s goals was to include as many students in the conversation as possible, she used the five talk moves to steer the conversation in a productive mathematical direction. She revoiced Dan’s comment, she elicited the reasoning of Dan, Brian, and Rita. She also used the other three talk moves during the conversation that followed. The result was that the students internalized the information Mrs. Holden wanted them to take responsibility for. They internalized this information because they all participated in meaningful conversation surrounding the concepts (Chapin & O’Connor, 2007; Vygotsky, 1934/1978).

Surely, preschool students are not ready for conversation exactly like the one described in the preceding paragraphs. However, it became clear in my study that the talk moves represented in this conversation were relevant for preschool math
conversation. In my study, I used rigorous qualitative methods to collect empirical data on how a teacher and students conversed about premath and mathematical topics in a preschool classroom.

In Chapin and colleagues’ model (Chapin & O’Connor, 2004, 2007; Chapin et al., 2003), teachers focused on drawing answers from the students as a group by moving from student to student. In the conversation moves demonstrated by the teachers in Franke and colleagues’ (209) study, the teachers are more hands-on with individual students, drawing answers from the individual by posing a series of questions to him or her. More research is needed to contrast these interaction styles and to determine which lead to best student outcomes. In the meantime, teachers may benefit from the variety of interaction strategies presented here. With knowledge of many types of conversational strategies, teachers can pick and choose based on individual student needs.

While probing questions were more open-ended than were leading questions, both conversational strategies were more closed-ended and teacher-centered than were the talk moves described by Chapin (204, 207) and Chapin and colleagues (2003). In the teacher-student interactions described by Chapin and colleagues, teachers avoided questioning any single student too closely, perhaps for fear of making the student feel wrong. It was strongly implied in that literature that the teacher’s emphasis should be on (a) encouraging more students to participate, (b) extending mathematical conversations, and (c) allowing students come around to the correct answer without undue pressure from the teacher. In the five talk move model proposed by Chapin and colleagues, more students participated. Further, conversations were longer.
As the previous examples illustrate, these different researchers two very different models of instruction, although both strongly emphasize the importance of discourse-rich instruction. Perhaps one model is superior to the other. More research is needed to answer this question. Perhaps teachers may benefit from having different models to make use of according to the needs of their particular groups of students. It is important to note that none of this research was conducted with preschool-age students.

An Elaboration of Talk Moves

In their study of one 10th-grade teacher in a city in Romania, Temple and Doerr (2012) found that both focusing and funneling were useful strategies depending on the teacher’s pedagogical goal in the lesson. These researchers reevaluated the classical IRE dialogue structure, interpreting the third move as feedback instead of simple evaluation. In their view, this third move could lead to a spiral of teacher-student interactions. In the following example, teacher and student utterances were categorized as interaction, response, or feedback moves. Having identified an expert math teacher for qualitative study, these researchers were able to study the intentional use of both focusing and funneling techniques described by Wood (1989). In addition, the researchers elaborated upon these two broad categories of teacher talk strategies by identifying more specific strategies that fell under each category. Temple and Doerr described 23 total talk moves that either teacher or student might employ in order to engage in extended, in-depth mathematical conversation. A description of a few of these talk moves follows.

The teacher in Temple and Doerr’s (2012) study was considered an expert for several reasons. She majored in mathematics in both undergraduate and graduate school,
she taught mathematics for over 25 years, she had taught many age groups (from fifth grade to college), she was a member of several mathematics organizations, and she had trained other teachers through a program under the Ministry of Education. These were only some of her qualifications. Further, it was noted that she taught in a decidedly constructivist fashion, putting the textbook to the side, opting to engage her students in conversation for most of her mathematics lessons.

The teacher studied by Temple and Doerr (2012) made use of multisemiotic discourse, employing spoken, written, symbolic, and pictorial representations in order to communicate mathematically. This multisemiotic discourse is characteristic of communication by professional mathematicians. Part of this multisemiotic discourse was the use of both everyday as well as formal mathematical language. This creation of hybrid space had been described by a number of researchers (Cobb, Boufi, McClain, & Whitenack, 1997; Moschkovich, 2002). Students required an expert teacher to help them navigate this multisemiotic, hybrid space. As a skilled interlocutor, this teacher built upon student language, moving them ever closer to more mathematical communication. Walkerdine (1991) called this process the \textit{matematization of language and reasoning}.

In this study, Temple and Doerr (2012) offered several examples from one extended episode in which the teacher asked the students to describe the Cartesian coordinate system, which she had described in a previous lesson. Now, the teacher wanted to activate the students’ prior knowledge by asking them to offer this explanation. In this example, the teacher built upon students’ everyday language,
effectively scaffolding (Ninio & Bruner, 1978) their development of more academic language. She made the workings of mathematical language more explicit so that students who were not yet fluent gain important exposure to this language. This act of bridging student language with formal mathematical language was in keeping with Vygotsky’s learning theory. The teacher located the students’ ZPDs through conversation and subsequently used her expertise to push their language development forward. In this example, Temple and Doerr (2012) noted after each utterance how they had categorized each type of teacher or student interaction move. “I” stood for initiation move, “R” stood as response, and “F” stood for feedback move.

T: What does the Cartesian coordinate system consist of? What is a Cartesian system of coordinates? (I—describing + defining)

S1: Two axes. (R)

T: Two axes . . . What kind of axes? (F—elicitation)

S2: Perpendicular. (RF—extension)

T: Two perpendicular axes. (F—recast + elicitation)

S1: That cross each other. (RF—extension)

T: That intersect in a point . . . (F—recast + elicitation)

S3: Called the origin. (Called the origin (RF—extension)

T (drawing on the board): Yes, this is called the origin. (F—reinforcement) (p. 293)

As illustrated in the above example, the teacher did not diminish or deny any single student’s contribution. Rather, she built on each one, carefully funneling the
answers so that the students themselves could find their way towards a complete mathematical definition of the Cartesian coordinate system. In the subsequent examples from the same teaching episode, the teacher used more of the twenty-three interaction moves that Temple and Doerr (2012) described in the appendix of their study.

The study conducted by Temple and Doerr (2012) is important because it elaborated upon the notion that teachers could support extended, inclusive mathematical conversations through the use of specific strategies. Strategies described by Chapin and colleagues (Chapin & O’Connor, 2004, 2007; Chapin et al., 2003); Franke et al., (2009); and Wood (1989) were all evident in the more elaborate descriptions of talk moves offered by Temple and Doerr. These studies could potentially help teachers who feel uncomfortable with math (Stipek & Byler, 1997) and do not know where to start in creating discourse-rich classroom environments. It may also help teachers who struggle to bring linguistic minority children into classroom conversations (Barletta, 2008).

Engagement in the right kind of mathematical conversations will aid in student math achievement (Cazden, 2001; Gee, 2009; Webb & Palincsar, 1996). The traditional IRE dialogue structure (Mehan, 1979; Sinclair & Coulthard, 1975) has not helped students gain conceptual understanding of mathematical strategies and procedures. However, revising this structure to include feedback instead of evaluation can help students to develop more mathematical ways of thinking and speaking (Temple & Doerr, 2012). To encourage children to offer open-ended mathematical explanation and communicate in a more authentic way, teachers can follow the talk move model offered by Chapin and colleagues (Chapin & O’Connor, 2004, 2007; Chapin et al., 2003).
Unfortunately, these researchers did not investigate mathematical conversations taking place in preschool settings. My study extended Temple and Doerr’s (2012) investigation to include a preschool classroom. The conversation strategies described in the previous studies were in fact relevant for the preschool classroom. The data also showed that novel conversation strategies were necessary to engage 10 preschool-age students during their math lessons.

Research on Math Talk in Preschool Classrooms

Chapin and O’Connor (2004, 2007); Chapin et al. (2003); and Temple and Doerr (2012) offered recommendations for conversational strategies that teachers could use to extend mathematical conversations and make them more inclusive than the traditional IRE format allowed. However, these researchers investigated middle school (Chapin & O’Connor, 2007) and high school (Temple & Doerr, 2012) classrooms. It was not clear that these recommendations can be applied to preschool classrooms in a straightforward way. In my thorough searches, I found only one study on teacher math talk in preschool classrooms (Klibanoff et al., 2006).

In this study, Klibanoff et al. (2006) found that all mathematical talk inputs on the part of teachers led to improved student math scores. In total, 198 children participated in this study. These children were drawn from 26 classrooms in 13 participating preschools and daycare facilities in and around the Chicago area. The researchers recorded teacher speech via audiotape during the school day in the middle of the year. Teacher utterances were coded for mathematical content. The types of math inputs that were coded by the researchers included cardinality, number symbols, counting,
conventional nominatives, calculation, ordering, nonequivalence, equivalence, and placeholding (Klibanoff et al., 2006).

Teacher inputs varied immensely. In the 26 classrooms observed, teachers provided mathematically-related inputs at a rate of 1 to 104 times. Children were tested with a mathematical battery before and after teachers were recorded for math inputs. Children were tested on items pertaining to ordinality, cardinality, calculation, shape names, understanding “half”, and recognizing conventional number symbols. For example, children were asked to point to a set of items that had more when one set had seven and the other had five. They were given simple word problems that required addition, given four possible answers to choose from.

Klibanoff et al. (2006) reported several findings. First, there was much variation in the conventional mathematical knowledge that children demonstrated at the age of four. Second, children from middle- and high-SES backgrounds demonstrated more mathematical knowledge than did children from low-SES backgrounds. Third, Klibanoff et al. (2006) found that the teacher math inputs varied dramatically from classroom to classroom. Finally, and most important for their study, they found that the amount of math talk provided by teachers was significantly related to children’s development of mathematical knowledge.

The findings of this study were relevant to the current discussion because it proved the benefits of teacher math talk for preschool-age children’s development of math knowledge. Because math talk had such a positive effect on these participating students, the findings encouraged teachers to try some of the conversational strategies
listed above with their own preschool students. The study of Klibanoff and colleagues (2006) led logically to the current study. Their study demonstrated that children benefit from math talk. Next, it remained to be seen what kinds of math talk students would benefit most from. It also remained to be seen whether all students benefited equally from different types of math talk. I investigated the specific types of math talk the participating teacher used with her preschool students. The teacher used many of the conversation strategies previously described. She also used completely novel methods to engage children in conversation. For example, she used manipulatives and visual aids to support conversation.

Sociomathematical Norms

This section connects the empirical findings described thus far with the theoretical and conceptual framework of this study. I reviewed findings from the fields of neuroscience and cognitive science supporting the idea that preschool-age children are ready to reason mathematically. I also reviewed the literature on academic language as well as that on linguistic differences in children that make development of academic language more difficult. Now, it is important to place these themes in the literature within a qualitative context. The work of Cobb and colleagues (Cobb et al., 1992; McClain & Cobb, 2001; Yackel & Cobb, 1996) on classroom microcultures served as the conceptual framework within which to understand all of this research as well as my study.

The classroom microculture (Cobb et al., 1992; McClain & Cobb, 2001; Yackel & Cobb, 1996) was the context in which sociomathematical norms and mathematical
language (van Kleeck, 2014) were constructed. It was also the context in which the teacher used conversational strategies (Chapin & O’Connor, 2004; O’Connor & Michaels, 1993; Temple & Doerr, 2010; Yackel & Cobb, 1996). Students’ linguistic differences had implications for how mathematical language and sociomathematical norms were constructed as well as for the conversational strategies that teachers were called to use (Chapin & O’Connor, 2007; Temple & Doerr, 2012; Wood, 1989). Cobb and colleagues investigated sociomathematical norms in kindergarten, first-grade, and second-grade classrooms. Research on this concept has been extended in recent years (Lopez & Allal, 2007) to include high school students. However, I found no study to date in which sociomathematical norms were investigated at the preschool level.

The Work of Cobb and Colleagues

In order to understand the meaning of this concept, it is important to revisit the rich qualitative data from which its current conceptualization emerged. As early as 1991, Wood, Cobb, and Yackel undertook a case study of a second-grade teacher in order to examine that teacher’s pedagogical practices. These researchers were specifically interested the ways in which the teacher verbally scaffolded (Wood et al., 1976) students’ mathematical construction of knowledge. In this study, as in their subsequent studies, the participating teacher struggled with how to balance a pedagogical agenda with the reform-oriented goal of letting children arrive at answers through a process of trial and error in conversation. The participating teacher supported students in developing mathematical speech and reasoning by starting with students’ own knowledge and unique ways of reasoning. She scaffolded them one step at a time as they appropriated more
formal mathematical ways of engaging in conversation. The National Research Council NRC (2010) and NCTM (2000) have both recommended that children take part in extended conversation. During these conversations, children should arrive at solutions on their own albeit with scaffolding (Wood et al., 1976) from the teacher. Thus, students should enjoy more agency in the classroom. For their part, the teachers’ roles should change from what they have been traditionally. Now that teachers were sharing their power with students so that students have agency in directing classroom conversation, it was not clear what classroom conversations should sound like.

In their 1991 study, Wood et al. chose a teacher who had previously taught in a traditional manner using a textbook but who demonstrated interest in learning a more constructivist, reform-oriented approach to teaching mathematics (NCTM, 2000). In several studies (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996) of similar methodological design, researchers helped teachers to bring their practice into closer accord with recommendations for discourse-rich mathematics instruction (Walshaw & Anthony, 2008).

Each week, after observation of mathematics classrooms in these similar studies (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996), the researchers sat down with the teacher to analyze the quality of the classroom interactions. They also assisted the teacher in identifying obstacles that stood in the way of students’ fuller participation in math talk. As a result of their study together, participating teachers were able to change beliefs and practices. Students made progress in developing strategies for solving mathematical problems. In their 1991 study,
Wood et al. worked with a teacher in a medium-sized midwestern city. This teacher had been teaching second grade for 15 years. She was recommended by her principal for professional development in the area of mathematical discourse. There were 20 children in the class from a range of socioeconomic backgrounds. All of the math lessons were videotaped during a full school year. In addition, field notes were taken during interviews with the participating teacher. Because of the time the researchers spent in the field, the data were rich and the theory that emerged from it robust. Initially, Wood et al. (1991) wanted to understand how children learned in a constructivist classroom. However, the focus soon shifted to include study of how the teacher changed her practice from traditional to constructivist. In the end, both student learning and teacher learning were investigated in this study. I will discuss student learning in this section, and teacher learning in a subsequent section on teacher talk moves.

The children in this classroom had much to learn about the norms in a discourse-focused classroom. In the procedure-based instruction that they were accustomed to, teachers usually described steps to follow to solve a problem, next gave them a problem, allowed them to work silently on these problems while the teacher corrected papers on his or her desk, and then told them the correct answer. Wood et al. (1991) helped the participating teacher introduce a constructivist style of mathematics instruction that emphasized three components: problem-based focus, pair work on problems, and whole class discussions of problem solution.

Wood et al. (1991) and the teacher made an effort to help students feel safe offering their answers whether they were correct or not. In fact, in departure from
traditional teaching, the focus was not on correct answer so much as it was on the thinking that went into the answers they found. The goal was for students to take responsibility for reconstructing their problem solving processes through conversation, explaining and justifying their work so that their peers could both understand and critique this work. The press for understanding was central, with both speakers and listeners struggling to understand the reasoning behind answers.

One finding that emerged early on in the study was that the teacher faced a conundrum in accepting students incorrect answers (Wood et al., 1991). The teacher faced this conundrum because, as per her pedagogical agenda, she wanted very much to communicate the correct answer as soon as possible. It is important to make note of this teacher’s conundrum to understand the context for the conversations that the students experienced subsequently. The teacher’s response to this problem early on was to use a discourse pattern referred to in the literature as funneling (Wood, 1989). In this talk move, the teacher used a student’s incorrect answer, then subjected the student to a series of questions which explicitly nudged the student towards the correct answer.

The following is a conversation from the beginning of the year in which the teacher was still accustomed to telling the students answers instead of allowing them to arrive at answers themselves at least part of the time. The funneling talk move is apparent in this example. The teacher showed the students an image of a pencil. Five paper clips were lined up end to end underneath the pencil. The teacher told the students that each paper clip was three centimeters long, then she asked how long the pencil was:

Chuck: I got 5 centimeters.
Teacher: You got 5 centimeters? How long is each one of these [clips]? Let’s take a look at this. If each one of these is 3 centimeters long, we have 3, 3, 3, 3, and 3. How much is that altogether? Karen?

Karen: 15.

Teacher: Do you agree or disagree?

Chuck: Disagree.

Teacher: Disagree? That’s alright, but there are three centimeters in each clip.

Chuck: There are 5 clips and that’s 5 centimeters.

Teacher: He is right. There are 5 clips, but how long is each clip? A 3 and a 3 and a 3 and a 3 and a 3 (measuring the distance on each clip with her hands). How long is that?

Chuck: 5 centimeters (Wood et al., 1991, p. 603).

During weekly meetings with the coresearchers, the teacher shared concerns, brainstormed new solutions, and received suggestions (Wood et al., 1991). The coresearchers discussed the above conversation during one such brainstorming meeting. They suggested to the teacher that students may not have an opportunity to struggle and arrive at their own answers when the teacher used this funneling technique. Not all researchers recommended against the funneling technique (Temple & Doerr, 2012). However, in the work of Cobb and colleagues (McClain & Cobb, 2001; Wood et al., 1991; Yackel & Cobb, 1996), the researchers encouraged teachers to extend conversations and to use strategies so that the students may make sense of their own mistakes. Then, they studied the student discourse that followed.
Over the course of the year, and because of this intervention, the teacher began to step back and allow the students to struggle through problems more on their own, enjoying rich conversation in the process (Wood et al., 1991). As a result of the teacher’s changing research style, student conversation and reasoning style also changed. As noted above, the researchers collected data on both teacher learning and student learning.

The following conversation reveals the progressive way in which students’ mathematical conversations changed in the participating classroom as the teacher met with the researchers over the course of the year. Norms changed so that students took more responsibility in deciding how to reason and talk about numbers. However, Wood et al. (1991) had not yet coined the phrase sociomathematical norms (Yackel & Cobb, 1996). Even so, here, the classroom environment was already characterized by the productive tension between students’ preexisting understandings and the formal conventions of larger mathematical society. In this conversation, a student the researchers called John posed a question about fractions. The class had been talking about fractions, and it occurred to him that he did not understand what 1/1 would mean. John asked the teacher to draw a circle, as this is how other fractions had been presented in previous lessons.

John: I have been thinking about this. What about if it was one and one? What would that be? Put a circle down.

Teacher: Okay. Put a circle down. (She draws a circle on the board, and then looks at John.)
John: Um like—what I’m thinking is one and one. (He points to 1/1 on the board.) How could you make this? (He looks at the teacher.)

Teacher: How could you show one and one, one over one?...

John: (Looks at the teacher and says nothing)

Ann: (Volunteers.) Just fill it all in.

Mark: Yeah! That’s just one piece.

Teacher: Okay. If we remember what a fraction tells us, John (He interrupts her excitedly.) But it’s the whole thing (he gestures with his hands, indicating a larger item), not just one piece (holds up one finger).

Teacher: So, how many can we fill in?


This example makes it clear that this student had been successfully socialized into the act of striving to find answers instead of waiting for the teacher to offer the correct answer. John was intrigued by a mathematical idea that he had confronted. He offered the problem without the teacher’s prompting. He initiated an exchange with the teacher. Next, in constructivist style, the teacher turned John’s question over to the group. The group successfully helped the struggling student to figure out the answer on his own. The teacher did not impose the correct answer. Rather, the group had negotiated the correct answer as well as the means of arriving at that answer. This co-construction of negotiation of mathematical understanding foreshadowed the concept of sociomathematical norms.
A notable finding in this study was that second-grade children were capable of learning nontraditional social norms for reasoning and discussing topics mathematically, with impressive results (Wood et al., 1991). In this study, the researchers were taking the first steps towards conceptualizing sociomathematical norms (Yackel & Cobb, 1996), but they had not arrived at that term yet. Significant for the current study, the researchers found that the changing norms introduced by the teacher changed children’s ways of interacting and reasoning mathematically together. Children truly internalized norms and then began to act upon them of their own volition, also adding their own flourishes. In this way, reorganization of mathematical thought became a process of negotiation by the group rather than an imposition by the teacher.

In the Wood et al. (1991) study, researchers found that students internalized norms introduced by the teacher and also reinterpreted these norms to make them their own. In a subsequent study, Yackel and Cobb (1996) confirmed that students co-construct norms with teachers Yackel and Cobb (1996) dubbed these norms, which specifically pertained to reasoning and talking about mathematical ideas, sociomathematical norms. They also confirmed that the process of internalization of norms led to students’ mathematical autonomy in the classroom.

Yackel and Cobb (1996) collected this data over the course of a full school year in a second-grade classroom. Once again, data were culled from video recordings of all mathematics lessons during the school year. Additionally, individual students were interviewed at the beginning, middle, and end of the school year. Finally, field notes were kept as well as copies of students’ written work. As in the previous experiment,
researchers worked closely with the teacher as coresearchers to introduce more discourse-rich classroom norms to the students. The researchers used strategies used in other project classrooms over the course of past years in the field. As a result, student thinking and discourse became more sophisticated as the year wore on. Students gained the ability to flexibly partition, decompose, and recompose numbers, using what appeared to be keen knowledge of the base-ten system. Further, students gained the ability to consider different solutions for the same problem at once, suggesting abstract understanding of the process of problem solving (Yackel & Cobb, 1996).

Yackel and Cobb (1996) focused on the concept of mathematical difference as a subject of normative thinking. When the teacher posed a problem, he did not stop asking for answers when one student arrived at the correct answer. Rather, he kept asking for different answers in order to ask multiple students to share their thinking, whether correct or faulty. He did so because two central tenets of constructivist mathematical instruction are to focus on thinking and not just on the correct answer as well as to include as many student voices as possible (NCTM, 2000). As a result of this practice, it was necessary for the teacher and students to negotiate what kinds of answers should be considered different. This negotiation process is illustrated in the following example. The teacher wrote the following problem on the board: $78 - 53 = \_\_$. The students were asked to describe their thinking in arriving at the answer.

Dennis: I said, um, 7 and take away 50, that equals 20.

Teacher: All right.

Dennis: And then, then I took, I took 3 from that 8 and that left 5.
Teacher: Okay. And how much did you get:

Dennis: 25 . . .

Teacher: Ella?

Ella: I said the 7, the 70, I said the 70 minus the 50...I said the 20 and 8 plus 3 . . .

Oh, I added, I said 8 minus the 3, and that’d be 5.

Teacher: All right. It’d be what?

Ella: And that’s 75 . . . I mean 25.

Dennis: (Protesting) Mr. K, that’s the same thing I said (Yackel & Cobb, 1996, p. 463).

Ella’s answer was too close to Dennis’s answer to merit sharing. However, she may have believed her answer different than Dennis’s because of her rephrasing of his answer. This discrepancy may have gone unchallenged had Dennis not been paying close attention to his classmate’s speech. In fact, he protested before the teacher had a chance because of his apparent thorough engagement in the mathematical conversation. The teacher had explained at the beginning of the year that students should only share their answer if it was different than the students who had shared ahead of them. However, the teacher did not explain explicitly what this mathematical difference should look like. Thus, it was up to both teacher and students to negotiate this sociomathematical norm. Dennis’s awareness not only of the correct answer, but also to the appropriateness of mathematical explanation, revealed abstract understanding of mathematical processes (Yackel & Cobb, 1996).
Another sociomathematical norm established in this classroom had to do with the experiential reality of the mathematical ideas shared. The teacher communicated the idea that students always connect discussions of mathematical procedure back to the real objects to which they refer. The following example demonstrated the extent to which the students had internalized this sociomathematical norm. Once again, the teacher communicated an idea, but did not put a finer point on the idea. It was once again up to both teacher and students to negotiate the meaning of this norm over the course of conversation. In this example, the teacher presented the following problem: “Roberto had 12 pennies. After his grandmother gave him some more, he had 25 pennies. How many pennies did Roberto’s grandmother give him (Yackel & Cobb, 1996, p. 469)?” The teacher then wrote on the overhead, after which the students explained how they came to their answers.

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12
+13
___
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Travonda: I said, one plus one is two, and 3 plus 2 is 5.

Teacher: All right, she said…

Rick: I know what she was talking about.

Teacher: Three plus 2 is 5, and one plus one is two. . .

Jameel: (Jumping from his seat and pointing to the screen.) Mr. K. That’s 20.

That’s 20.

Jameel: Ten. Ten. That’s taking a 10 right here…(walking up to the overhead screen and pointing to the numbers as he talks.) This 10 and 10 (pointing to the ones in the tens column). That’s 20 (pointing to the 2 in the 10s column).

Teacher: Right.

Jameel: And this is 5 more and it’s 25.

Teacher: That’s right. It’s 25.

In this example, Travonda refers to a number in the tens column as if it were a number in the ones column. She may have understood that the number she referred to as “one” was a ten. She may then have been referring to it in shorthand by calling it a one. However, her classmates considered this shorthand inadmissible. The ones in the tens column referred to pennies, and thus her classmates insisted her speech reflect this reality. The sociomathematical norm for admissible speech was thus established as taken as shared, to use Yackel and Cobb’s (1996) term. The meaning of taken as shared was not at all clear in the writing of Cobb and colleagues (2001); however, it appeared that a norm was taken as shared when more than one member of the classroom microculture agreed upon it. Thus, an incident of a taken as shared may be very important in understanding the emergence of sociomathematical norms.

Other Research on Sociomathematical Norms

Lopez and Allal’s (2007) research on sociomathematical norms in two third-grade classrooms picked up where Cobb and colleagues (Cobb et al., 2001; Yackel & Cobb, 1996) ended. Lopez and Allal adopted their concepts of the classroom microculture and taken as shared and extended their conceptualization of sociomathematical norms.
Yackel and Cobb (1996) stated that the expectation that students should explain their thinking is a more general academic social norm, and not special to mathematics. By contrast, Lopez and Allal (2007) argued that this norm takes a certain form in mathematics class and should thus also be considered a sociomathematical norm. The principle aim of their investigation into two classrooms was to extend the utility of the concept sociomathematical norm as a means of investigating mathematical practices that are shared in classroom microcultures.

The two teacher participants, whom Lopez and Allal (2007) called Paula and Luke, demonstrated some similarities and some distinct differences in their teaching styles. The qualitative data were coded for the emergence, subsequent elaboration, and process of taking as shared for sociomathematical norms in each classroom. Next, they looked at how these norms regulated classroom practice. Two sociomathematical norms were consistent in both classrooms. The first was that students were required to explain their thinking to the class when they offered a solution to a problem. Second, once a method of reasoning had been proposed, students should refrain from contributing if their method of solving the problem had been the same. This finding called to mind McClain and Cobb’s (2001) examination of their teacher’s taken as shared criteria for mathematical difference.

The classrooms differed in two important ways (Lopez & Allal, 2007). In Paula’s class, students were encouraged during discussions to rephrase other students’ explanations in their own words. They could also take on a classmate’s incomplete explanation and elaborate upon it. In Luke’s class, students were expected to approach
one another’s contributions differently. Luke encouraged them to critique one another’s explanations in terms of their problem-solving efficiency. Paula, much like the teacher in Cobb et al.’s (1997) study was interested in hearing all the possible solution strategies her students could generate. By contrast, Luke preferred for his students to learn how to hone in on the most efficient solution strategy. These major differences made for a number of smaller differences in sociomathematical norms.

Lopez and Allal (2007) found through their investigation that the concept of taken as shared (Cobb et al., 1992) was particularly useful for understanding how classroom conversation progressed in complexity and abstraction. This term, which was a bit obscure in Yackel and Cobb’s (1996) work, gained clarity in Lopez and Allal’s (2007) study. As sociomathematical norms were interpreted in specific instances of conversation, their situated meaning became taken as shared. The researchers argued that these taken as shared instances serve as a common reference or jumping off point for subsequent conversations. As these common reference points, or objects of reflection (Cobb et al., 1997) accumulated, the mathematical reasoning of the whole group grew in complexity and abstraction. Without such references, Lopez and Allal (2007) surmised, the conversations could never transcend the specifics of any one problem solution.

Lopez and Allal (2007) confirmed the usefulness of the framework established by Cobb and colleagues (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; Yackel & Cobb, 1996). Through their own investigations, they found individual students internalized the reasoning that had initially been negotiated in the group setting. The researchers thus posited a transactional relationship between the
social and the cognitive. They found the concept of taken as shared especially useful. Because reasoning that occurred within an individual’s mind could never truly be captured, it was necessary to focus on verbalizations or other expressions of thought in order to examine it. It was possible to observe when members of a group come to a verbal consensus on how to talk about a shared experience. It was also possible to observe when individual students demonstrated ownership over a taken as shared idea. The researcher’s task aside, the data also suggested that classroom participants had no way of reasoning together until individuals engaged in verbal exchanges about ideas. As members of the classroom microculture came to agree upon ways of talking, they also come to agree on ways of thinking (Lopez & Allal, 2007).

My study made use of two important concepts from the research described above (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb, Stephan et al., 2001; Cobb & Whitenack, 1996; Lopez & Allal, 2007; Yackel & Cobb, 1996). Specifically, I found that the concepts of classroom microculture and sociomathematical norms were useful for describing the scenarios that unfolded in my own study. I found the concept of taken as shared to be less useful. The participants in the classroom I studied co-constructed norms for reasoning and thinking mathematically as a group. These norms led to the development of a classroom microculture that was both unique and productive in shaping individual students’ math knowledge.
CHAPTER 3

METHODOLOGY

My research was guided by an overarching research question and three sub-questions. My principal question pertained to the role that sociomathematical norms played in a preschool classroom. As part of this larger question, I asked how sociomathematical norms were constructed in a linguistically diverse classroom. I also posed the question of what obstacles existed to the co-construction of these norms. Finally, I asked what conversation strategies the teacher used to support the co-construction of sociomathematical norms.

Research Design

The purpose of the current study was to explore the ways in which one preschool teacher promoted the development of mathematical conversations in a preschool classroom. I explored the ways in which teacher instructional decisions permit linguistically different students to access mathematical instruction. I made use of the ethnographic research method as I embarked on this study, drawing on elements from the long tradition of ethnographic research, and specifically from the somewhat shorter tradition of ethnographic research in education.

The 1960s brought the ethnographic method to bear upon problems in education in a more meaningful way than ever before. Since the beginning of the nineteenth century, many had extolled qualitative research as a means to address suffering in
society. However, it was not until the 1960s that researchers began vigorously applying this approach in school environments. During that era, citizens and politicians demanded to know what disadvantaged students actually experienced in schools. These new socially engaged researchers wanted to ensure that the informants’ own perspective on their often unjust experiences be put under the spotlight. Giving voice to the voiceless, the ethnographers of this period captured the public’s attention with their unique and sophisticated approach. The field of sociology, now turning away from the structural-functionalist view which had dominated it for twenty years now turned towards ethnomethodology. Others identified with the symbolic interaction school. The emergence of these new sociological schools was evidence of the sudden surge of interest in qualitative methods. This new interest was complemented by the publication of important books on theory and method (Bogdan & Biklen, 2007).

The education researchers of today owe a direct debt to the often political ethnographers of the 1960s. Now, the ethnographic study plays just as important a role as ever in detailing the lived experiences of marginalized students in the classroom. For example, socially-engaged qualitative research has offered a glimpse into the reasons why achievement gaps exist. Numerous quantitative studies had already demonstrated that achievement gaps existed between children of different economic or ethnic backgrounds, but they offered no potential explanation for why this was the case. Quantitative studies offer certain types of information, but how are we to understand the ways in which the achievement gap is perpetuated? Careful ethnographic study has shed light on the cultural construction of the achievement gap in disparate settings.
In parallel ways, famed qualitative researchers like Wolcott (2008) and Spradley (1979, 1980) fleshed out the art of ethnographic research by delineating and illustrating examples of how to collect and interpret qualitative data. As an extension of the social justice concerns of the 1960s, Wolcott explored avenues to engage in intensive personal interactions with informants in context. Close personal knowledge of participants’ experiences allowed him to gather convincing evidence to support sociological conclusions. Together, Wolcott and Spradley played important roles in expanding the uses of the ethnographic method. Wolcott, an educator and ethnographer, pioneered the area of ethnography in educational settings. Spradley, with his signature ethnosemantics, a technique for identifying the categories of thought in a culture, encouraged ethnographers to study settings that were close to home instead of those that were far away and exotic. It is important to note that Wolcott (2008), although praising Spradley for offering a technique that was sufficiently methodical to make it accessible to new researchers, expressed criticism of Spradley’s technique, claiming it to be slightly rigid for the purposes of true ethnographic research.

Researchers like Wolcott (2008, 2010) and Spradley (1979, 1980) helped to establish the legitimacy and specificity of ethnography as a research method (Bogdan & Bicken, 2007). Researchers using this method were to thoroughly immerse themselves in the culture that was the object of study. Wolcott (2008) emphasized that in order to immerse one’s self in the study of a culture, one must travel to the context in which the culture exists. The researcher must also make every effort to break out of the mental constraints imposed on him or her by the data others have gathered about the culture. In
this way, the researcher can uphold the original philosophy of early ethnographers, which was to understand the culture in that culture’s own terms (Wolcott, 2008; Spradley, 1979, 1980). Finally, Wolcott emphasized the role of the researcher as instrument, suggesting that the ethnographic method should be characterized by a sanctioned open-endedness so as to permit freedom of what he calls mindwork, or the type of detailed analysis that can only be conducted qualitatively. Wolcott suggested that this mindwork can never be meaningfully reduced to mechanical steps for the researcher to follow. The rule of ethnography was thus for the researcher to continuously generate his or her own rules for the analysis of the culture. The scientific nature of the method lay precisely in its ever-emerging form of analysis.

In the case of my study, I shed light on how students appropriated mathematical communication skills simultaneously with their appropriation of mathematical reasoning skills. Research confirmed the fact that students co-constructed and appropriated the norms of the classroom microculture as they verbally negotiated the meaning of mathematical problems, solution strategies, and the significance of these strategies (Cobb & Bowers, 1999; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Cobb, Stephan, McClain, & Gravemeijer, 2001; Cobb & Whitenack, 1996; Lopez & Allal, 2007; Yackel & Cobb, 1996). I added to this body of research by examining how 10 preschool students, including two linguistic minority students, accessed opportunities to take part in this negotiation process. Research by van Kleeck (2014) suggested that in many classrooms, the mathematical meanings in the mathematical microculture ultimately came to be authored by those students who were more privileged when they arrived due
to their different home experiences. These students were privileged in that they already spoke academic language, having been exposed to it at home. As a result, they spoke more like the teacher from the outset. The teacher favored them because of some unconscious or conscious expectations of how children should speak. By using ethnography to understand the experiences of students of different backgrounds in solving math problems, I situated myself in the particular qualitative tradition established in the 1960s in the United States, which was extended by ethnographers of Wolcott (2008) and Spradley’s (1979, 1980) ilk.

While aspects of many forms of qualitative research were present in this methodology, the ethnographic model was most appropriate for the purposes of studying the culture of a single classroom. Other forms of qualitative research include grounded theory, phenomenology, case study, and narrative inquiry (Lichtman, 2013). Most importantly, the method of the current study made use of the grounded theory approach espoused by Glaser and Strauss (2008). In contrast to Wolcott’s (2010) emphasis on open-endedness and a kind of mindwork that cannot be described, Glaser and Strauss described a method in which the researcher quickly developed the beginning of a theory with the very first data collected. Glaser and Strauss (2008) described data collection with the phrase theoretical sampling.

Theoretical sampling referred to the process of data collection for generating theory. Each time researchers returned to the field, the data they had previously collected determined what they would collect next. Each time they collected data, they used it to form a tentative theory. Next, they returned to the field to test this theory. Glaser and
Strauss (2008) stated, “This process of data collection is controlled by the emerging theory” (p. 45). It should be noted that while I made use of theoretical sampling during the data collection process, a great deal of the data analysis and final development of local theory took place after the data collection had come to an end.

The purpose of theoretical sampling, according to Glaser and Strauss (2008) is to define concepts or categories that fit the data collected. Immediately after the very first data collection, the researcher is to form conjectures about how to define these concepts and categories. Then the researcher will return to the field with the goal of minimizing and maximizing these definitions, with the goal of observing them under different conditions. The researcher might find that the categories varied under different conditions. Each foray into the field is a search for was theoretically relevant data. Each search is purposeful. The goal is always to develop the existing categories. The researcher is to “compare everything comparable”.

Wolcott (2008, 2010), Spradley (1979, 1980), and Glaser and Strauss (2008) do not disagree necessarily. Wolcott also described a method in which the researcher makes rules for data analysis over the course of the research. However, Glaser and Strauss described in more detail the driving force behind the theoretical sampling of data. As noted, the researcher is to form a theory as soon as the first data are collected. Groups or categories of information are coded in this data. The next data collection, or theoretical sampling, is conducted with the intention of fully maximizing and minimizing these categories as well as their relationships to one another. This development of categories and their relationships contributes to the grounded theory that emerges as a consequence.
This study investigated the unfolding negotiation of sociomathematical norms through premath and mathematical conversations in a single prekindergarten classroom. Creswell (2013) described the ethnography as ideal for studying the shared social practices of a culture. Aspects of narrative and phenomenology were present as I conducted interviews with the teacher in an attempt to capture the emic perspectives of their experience, before filtering these through my etic, scientific perspective of the data (Creswell, 2013). Finally, a dimension of the case study model was present in that I selected a single classroom, a case bounded by time (a semester) and space (the classroom). However, the ethnography approach was the most suitable label for the present study because of the situation of my analysis in that specific qualitative tradition. The interviewing style I used, for example, was based on the technique described by celebrated ethnographer Spradley (1979, 1980). Furthermore, the overarching goals of cultural interpretation, I took from Wolcott (2008), who was himself a self-described ethnographer. The ultimate aim of my study was to capture the shared cultural meanings of activities and events for the members of the classroom. This aim was truly ethnographic in nature.

There were two modified aspects of the ethnographic nature of the study. Unanticipated by me, the teacher felt comfortable changing her practice because of her participation in my official dissertation study. She felt her participation in my study protected her from outside criticism of her changes to her practice. In other words, the fact that she was participating in my study influenced the teacher’s practices. This fact prevented me from capturing the behavior the teacher would have engaged in
without my influence. Another modification to the strictly ethnographic design was a single instance of feedback I gave to the teacher, whom I have called Chen. In this instance, during an interview, Chen was lamenting her lack of strategies to orchestrate productive mathematical conversation. At that moment, I described to her the strategies that I had observed her using which also happened to be described in relevant research. Although I simply pointed out this fact, this subtle intervention changed the track of the study. It changed the direction of the study because Chen then asked to see the research in which her strategies were described. I shared a single study with her, but this study made a big difference for Chen’s practice. She began using not just one, but all of the strategies described in the study. Additionally, this research emboldened her to develop her own conversation strategies.

Creswell (2013) noted that an ethnography takes as its subject of study the “behaviors, beliefs, and language of a culture-sharing group” (p. 90). I aligned myself with the researchers who expanded upon this ethnographic framework to include the shared cognitive behaviors of the group, such as reasoning, problem solving, and argumentation. Because I viewed these behaviors as part and parcel of the shared practices of the classroom microculture, the ethnography was the appropriate approach for the study in spite of the limitation of my influence. Creswell (2013), too, noted that the goal of an ethnography is to create a holistic portrayal of a culture. I would argue that a comprehensive picture of a culture should include its reasoning and other cognitive practices which are condoned and disseminated among the group. Wolcott’s (2008) description of what questions should be answered in an ethnography includes what the
people in the group have to know in order for the system to work. In this study, this knowledge included beliefs, ideas, and an unfolding understanding of how to reason.

Research Participants

Finding the appropriate classroom for these research questions was not a straightforward process. My goal was to identify a preschool teacher who would be interested in discussing lessons with me as a means of reflecting upon the mathematical conversation that took place in the classroom. I had to find a teacher who believed in incorporating conversation into his or her math lessons. In this sense, Chen seemed to fit the requirement perfectly. She was ready to reflect on her practice and even to change it. My goal was to observe these changes without necessarily propelling them.

It was also important to my study that this teacher be willing to discuss the linguistic differences (O’Neal & Ringler, 2010) of students and how he or she planned to negotiate these differences as the whole class talked about mathematics. Although Chen initially expressed some interest in participating as a coresearcher, she changed her mind as the study commenced. She chose instead to focus on designing the best possible instruction for her students. Her practice was influenced by her participation in my study, but she did not end up acting as a coresearcher with me.

Cobb et al. (1997) studied the work of a first-grade teacher who used their feedback as she continued to refine her instruction throughout the study. The teacher’s instructional approach underwent a fascinating process of transformation as a result of these ongoing conversations. This particular study illuminated just how much potential reform math practices have in changing the way teachers and students communicate in
the classroom. This type of study has not been replicated in recent years, nor has any such study been conducted at the preschool level. For this reason, I wanted to conduct the current study as coresearcher with Chen. However, our only interaction that approached the relationship of coresearchers was that in which she expressed her frustration with me about her inability to engage her students in deeper, extended conversation. In this instance, I pointed out to her that she was using two research-based conversation strategies: revoicing and recasting. This seeming small act of coparticipation had far-reaching effects on Chen’s practice.

Becker (1998) noted that a researcher must make shrewd decisions when it comes to sampling. A researcher should not and cannot study everything. In selecting participants, my goal was to bound a single, small ethnographic case so that I could understand the setting and participants’ experiences deeply. My goal was for this deep study to render thick, rich description (Geertz, 1973) of mathematical conversations in a single preschool classroom. My goal was not, ultimately, to select a generalizable sample. Instead, I selected participants whose experiences together would tell a powerful story. In doing so, I captured both successes and challenges to offer food for thought to researchers and teachers. The verbal exchanges in a single preschool classroom will never yield enough data to permit generalization about other preschool classrooms. However, as subsequent researchers continue to collect more data in this vein, they may begin to make a case for general conclusions (Bogdan & Biklen, 2007).

In light of Becker’s (1998) words, I have considered my own professional and scholarly interest in verbal communication and made the determination to find an
interactive preschool classroom in which to establish my presence. This selection did not
tell me about what goes on in any other preschool classroom (Geertz, 1973). Many
questions about mathematics education at the preschool level remain open for others to
explore. My sample—one teacher and 10 students in a single preschool classroom—
revealed what kinds of mathematical conversations occur in that environment alone.
Although I was curious about what occurs in Georgia prekindergarten classrooms, it was
more likely that I would observe the kinds of nontraditional teacher-student and student-
student interactions in a private school classroom.

I had another reason for selecting a private preschool classroom for study. I,
myself, had taught at the early childhood level exclusively in private schools for eleven
years. Because of my experience in private preschools, I felt I was more sensitive to the
nuances in interactions in this type of setting, and also more aware of my biases there.

In this decision, I would like to contrast myself with the 19th-century
ethnographers who “went native” and insinuated themselves into a culture of study so
deeply that they lost any sense of separation from it (Bogdan & Biklen, 2007). Later
social scientists harshly criticized the naivety and presumptuousness of these early
ethnographers who, in spite of their close physical proximity to indigenous cultures,
ultimately drew problematic conclusions about these cultures’ primitivity. These
accounts romanticized and “othered” participants and painted them as lacking in the
sophistication or banality of the researchers’ own cultures. I hope to have escaped the
magical thinking inherent in this practice of “going native” by studying a setting that was
more familiar to me.
Difference lies in our perspective. No matter how similar or different we are from our participants, we are always, in fact, different. The idea that a researcher must travel far from home, or investigate a very unfamiliar setting, in order to observe difference is outdated and politically problematic. I feel certain that an unfamiliar classroom in my own preschool was sufficiently difficult for me to understand. Furthermore, I was under no illusion that the participants in this setting were special in some magical, othered way. It is “validity-enhancing” for a researchers to stay in a setting where they can be certain of their inability to be objective. As a social constructivist (Creswell, 2013; Denzin & Lincoln, 2011), I reject the positivist belief in objectivity, preferring instead a model of reality as constantly under construction by society. This view of perspective as constructed allowed me to insert myself in the study at one point in which I pointed out to the teacher that she was using two research-based conversation strategies. I knew that, in spite of my best efforts, I was influencing her by my mere presence in the room. Thus, it was not a big jump to give her a small bit of feedback on her practice. In my own qualitative report, my goal was to narrate the story of how I co-authored the reality of classroom experiences with my participants over the course of two semesters.

Ultimately, I identified one teacher in my own school who was progressive enough to converse rigorously about her practice during our interviews, particularly concerning the use of conversation as an instructional tool. I have called this teacher Chen. Chen was Taiwanese-American. She had been teaching in this school for eleven years. It was her first and only teaching position. Two of the students in her group were ethnically and also linguistically diverse, but all came from upper-middle- to upper-class
homes. Chen’s ten students were also participants in my study. Chen and her co-teacher taught math once or twice a week, breaking the class into two groups of ten students each. I studied Chen and her group of 10 students. I videotaped all of the math lessons that Chen taught during the spring semester of 2018.

Although these teachers and students were a part of the school where I taught, this was not merely a convenience sample. Chen especially voiced a strong interest in improving her ability to orchestrate productive mathematical conversations in her classroom. When I described the nature of my study, Chen stressed her belief that she fit the description of the type of teacher I was hoping to sample.

In the beginning, Chen seemed eager to act as a coresearcher with me. She welcomed my feedback: “If you have ideas for how we can teach math better, please share with us,” Chen expressed to me. “We’re always trying to teach math in more creative ways.” However, once the study began in earnest, Chen backed away from this role. She found it more productive on her end to focus on planning lessons by herself. However, she asked me to provide her with a piece of research on the single occasion which I have described. When she asked me for ideas on how she could engage students in more productive mathematical conversation, I was careful not to present novel ideas. I wanted to stay true to the ethnographic tradition and gain information from the emic perspective. Thus, I was careful to point out only what I had already observed Chen doing. I pointed out that she had used two research-based conversation strategies. I did not introduce any new information or research that may have led her down a radically different path. At the same time, my slight intrusion did influence her plans for
instruction. Chen asked to see the research I was referring to and this research inspired her to change her practice further. It is hard to know if Chen would have discovered this research had I not shared it with her upon her request.

Data Collection

When planning for data collection, it is important to remember that the finished product of an ethnographic study should detail a prolonged dialogue between research and participants (Bogdan & Biklen, 2007). Only through such prolonged dialogue can culturally shared meanings be apprehended. Ethnography is a means of collecting empirical data that involves intense personal interactions with informants. Although the method is romanticized by some and declaimed as impossible by others, careful reading of previous ethnographic literature will reveal that the method is characterized by specific conventions and techniques that can be learned and faithfully applied (Spradley, 1979, 1980; Wolcott, 2008).

The method of data collection is the aspect of ethnography that marks it as distinctive. Many borrow from ethnographic techniques, but for the purposes of this study, I defined the pure ethnography in terms of how Wolcott (2008) and Spradley (1979, 1980) developed and defined the method. Wolcott argued throughout his oeuvre that the primary purpose of the ethnographic study was to reveal the underlying meanings of cultural objects, practices, and ways of being. Ethnography predates all other qualitative research methods. The many derivations of the ethnographic study such as narrative study, phenomenology, grounded theory, and case study (Creswell, 2013) all
borrow from its techniques. However, the true ethnography takes cultural meaning as its primary focus.

Wolcott (2008) insisted that ethnography was a “way of looking,” but he and Spradley (1979; 1980) described more specific components of the work as well. One defining characteristic of Wolcott’s (2008) style of ethnographic research was his imperative that the researcher visit the site in question rather than pulling participants out of their setting for interviews or observation. A second hallmark of ethnographic research is the use of multiple data collecting techniques, with fieldwork (observations and interviews) forming the centerpiece of the method. The key to fieldwork that is truly ethnographic and not simply ethnographically-influenced is prolonged time in the field (Wolcott, 2008). The level of intimacy with the setting that ethnographic research requires will be difficult to achieve in the space of one semester.

The weaknesses of this method are mediated when the researcher remains for a long period in the field (Wolcott, 2008). If participants change their behavior due to the presence of the researcher, over time it should become difficult for them to maintain this unnatural behavior. If the participants alter their behavior in response to some other external factor while the researcher is visiting, prolonged experience with the participants will ensure that the researcher does experience their more typical behavior for at least part of the time. My prolonged time in the field thus made up for any undue influence I had on the participating teacher. It was vital that I located a preschool classroom whose math lessons I could visit at least every week over the course of the semester-long research period.
Fieldwork consisted of both observations and interviews, which included casual conversation (Bogdan & Biklen, 2007; Wolcott, 2008). Wolcott noted that during observations, many factors might influence the extent to which the researcher participates in the activities he or she will observe. The researcher may choose to participate extensively or not at all. In order to stay true to the overarching goal of ethnographic research—that of remaining flexible and open to serendipitous discoveries (Spradley, 1979, 1980)—I did not make decisions regarding the extent of my participation before entering the setting. As I gained knowledge of setting and participants, I made decisions regarding the extent of my participation. In the ethnographic tradition, researchers make such decisions in order to put participants at ease. Rapport with participants enables researchers to more easily access the data they aim to collect. I was at an advantage in this regard because I had already established easy rapport with my adult participant as well as with many of the children in the class. I followed the teacher participant’s lead in how much I participated in the study. In fact, I was a detached observer except in the single instance in which I gave a piece of feedback and the teacher requested that I share one study with her.

Because the teacher participant did not want to be a co-researcher, during interviews I asked the teachers questions that were as open-ended as possible. This type of questioning allowed the teacher to take the lead in describing her practices and her efforts to incorporate more conversation during lessons. The overarching goal of the interviews with the teacher was to produce rich and in-depth ongoing dialogue that would
unearth her true motives as a teacher in the classroom and her hopes for creating a more discourse-rich community.

Official interviews were semistructured, but interview data also included any and all casual conversations that took place in the classroom during participant observations (Wolcott, 2008; Spradley, 1980). I made an effort to keep conversations open-ended. However, in the spirit of Glaser and Strauss’s (2008) theoretical sampling, I drew questions directly from my observations of the teacher and students and the grounded theory that began to emerge from these sources. Although I was interested in how the participants in the classroom microculture co-constructed sociomathematical norms through verbal discourse, I thought it best not to ask numerous specific questions. Spradley noted that questions that appear to value some ideas or events more than others can lead to closed-ended conversations.

To design interview questions in an ongoing manner, I consulted my notes from preliminary observations and endeavored to understand the whole context of the teacher’s math lessons. In addition to observations on the general classroom environment, the materials, the parental presence, the administrative presence, the teacher’s general behavior and the students’ general behavior, and all other contextual factors, my special focus was on the verbal exchanges between the teacher and students and between students and other students.

I kept a keen eye out for cultural and linguistic difference in student contributions to mathematical conversation and for the types of contribution the teacher accepted and approved of. Previous research on mathematical language (van Kleeck, 2014) and on
general classroom discourse suggested that cultural and linguistic differences in students’ verbal contributions could lead to unequal access to classroom conversation. I was curious as to whether or not this would be the case in a preschool classroom. However, I was prepared to bracket (Patton, 2002) these initial interests in order to remain sensitive to the unique data that would emerge in this context. In the spirit of the seminal ethnographic research of the 1960s, I made every effort to report the teacher’s and students’ perspectives on these classroom proceedings. During interviews, I followed up on observations I had made in the classroom by asking the teacher to describe classroom discussions in his or her own words. My preliminary questions were established: I asked Chen what did and did not go well in classroom discussions and how she would use this reflection to move forward in subsequent lessons. Subsequent interview questions grew organically out of observations of teacher-student and student-student conversations. My ethnographic goal was to allow the teacher to tell me what is important for me to know. Wolcott (2008) reminded the qualitative researcher that questions are always culture-specific and that researchers must endeavor to gain awareness of cultural influence on their inquiry as they enter the culture of their participants. Spradley (1979) wrote that “an ethnographic interview is a particular kind of speech event,” (p. 55). Spradley thus prompted researchers to take an ethnographic view of their own ethnographic method. The culture of the examiner is always interacting with the culture of the examined. Spradley goes on to write that the true ethnographic interview, as a speech event, shared features with casual, friendly conversation. Maintaining the semblance of friendly conversation supported the development of rapport. Thus, it was important to
spend time in informal greetings and small talk, to keep the conversation open and free of a too-explicit purpose, to refrain from asking repetitive questions, and to otherwise maintain a sense of ordinariness in the conversation.

The methodological challenges of this study were manageable. Questions about sociomathematical norms were suitable questions for an ethnographic study (Goos, 2004; Lopez & Allal, 2007; McClain & Cobb, 2001; Yackel & Cobb, 1996). Ethnographies explore the practices of a culture, first and foremost (Bogdan & Biklen, 2007). I collected data in such a way as to be true to this overarching goal.

My findings were significantly different in nature when contrasted with these previous studies because I conducted my study in a preschool classroom, whereas previous researchers interested in sociomathematical norms examined K-12 classrooms. I gathered data that extended knowledge of these norms as they were constructed in the preschool classroom. Cobb and colleagues did not synthesize research on linguistic differences (O’Neal & Ringler, 2010) with their own work. I examined how the concept of sociomathematical norms was complicated by the linguistic differences of two participants. I wondered if the norms of this classroom would ultimately reflect the cultural hegemony of the linguistic majority students in the classroom. I also wondered how these norms regulated and/or inhibited the learning of linguistically different students. I discuss my findings regarding these questions in Chapter 4.

Data Analysis

If the driving purpose of ethnographic data collection is the discovery of cultural meanings, the same fundamentally anthropological purpose also drives the data’s
analysis. Wolcott (2008) wrote that the first things the researcher should record are vignettes which reveal specific behaviors. Next, the researcher should look for patterns of behavior that emerge from these fast-accumulating vignettes. As numerous varied patterns of behavior emerged from the fieldnotes, the relationships between these patterns, as well as the direct verbal testimony of participants, would reveal the meanings these behaviors hold for participants.

Wolcott’s (2008) work represented the anthropological tradition of inquiry into socially shared behaviors. The specific behaviors I observed included all forms of interaction: physical, verbal, and written. I reported on the whole context in which these interactions occur so that their situated meaning would become apparent. This search for patterns in socially shared behaviors as well as their meanings drove the progressively focused analysis of the study.

In the case of mathematical conversations in the preschool classroom, so little was known that the ethnographic method was eminently called for as a first step toward an understanding of this social phenomenon. I was dedicated to the implementation of the ethnographic method of study in order to open up lines of inquiry for further research in this area (Wolcott, 2008). Bogdan and Biklen (2007) echoed Wolcott’s contention that previously published literature can and should enrich analysis. These researchers described a careful balance between the use of concepts from substantive literature in the area and a sensitivity to occurrences that took place over the course of the study. A true ethnographer was aware of his or her purpose in approaching the study, but was also
responsive in allowing analysis to proceed from unexpected data. This was a balancing act that could be achieved.

I brought to the data collection process the goal of identifying the discourse practices that were crucial to the production and reproduction of mathematical culture in a single classroom. One assumption that I brought from theoretical reading is that group discourse was internalized by individuals, so that group discourse was internalized to become individual logic (Vygotsky, 1934/1978). However, it is unknown exactly how internalization takes place or what factors may problematize the process of internalization of mathematical language. It is also unknown how this process occurred at the preschool level. These questions led to identification of certain themes over the course of data analysis. My aim was to collect and analyze data with as open-minded a view as possible within the above parameters (Bogdan & Biklen, 2007; Wolcott, 2008).

While Wolcott (2008) noted that ethnographers use a wide variety of methods in their analysis, Creswell (2013) provided a fairly clear-cut outline of the general practices of many qualitative researchers, Wolcott included. Most researchers, according to Creswell, progressed through the same stages of data analysis. They collected data, organized it, coded it, condensed codes, and then went through codes to reduce data into intelligible themes. It was important to note, however, that qualitative researchers did not progress through these phases in lock-step fashion, but rather moved forward and backward through the stages in an iterative process.

The process began when the researcher collected data, then tentatively organized and reflected upon it. The organization phase entailed reading the whole initial database
in order to get a feel for its meaning, dividing it into segments (phrases, sentences, or interactions), and then labeling these segments according to tentative categories. The researcher next used this provisional organizational system to generate reflections. These reflections, kept in memos, guided the manner in which the researcher made choices about subsequent data selection.

This process was iterative in that, as the researcher entered new data into the organizational system, it would either fit well or it would change the manner in which the researcher organized and labeled the whole. Thus, each new phase of data collection would force the researcher to alter the organizational system. Incoming data might inspire the researcher to divide the data into different units than he or she did originally. As data accrued, the researcher also gained a cumulative understanding of its meaning which would enable him or her to create visual or written representations of the database (Creswell, 2013). My own data analysis followed these patterns.

Creswell (2013) suggested limiting the number of smaller categories to 25 or 30 codes. This list of codes would eventually be connected thematically and thus reduced to five or six that will enable deeper analysis and final presentation. My own codes were slightly more numerous than what Creswell suggested. I began data analysis with close to 40 codes. Ultimately, the organization of data into themes renders them accessible to the researcher’s analysis. If not organized in the end into five or six major themes, the task of understanding the interrelationships between the pieces of the whole would be hopelessly complex. The researcher might satisfactorily understand the interrelationships between five or six major themes. Moreover, these larger interrelationships might
provide glimpses into the more nuanced relationships between bits and pieces in the findings. My own data analysis resulted in five major themes, just as Creswell recommended.

I did not plan to use any of the research concepts from my literature review as prefigured codes. However, as I coded for what emerged organically from my data, I memoed about connections I saw between my codes and the relevant substantive literature, which included concepts such as mathematical language (van Kleeck, 2014), linguistic difference (Lagrand & Reid, 2000; O’Neal & Ringler, 2010), and sociomathematical norms (Yackel & Cobb, 1996). These connections inspired me to limit my study, since limiting the study was a must (Glaser & Strauss, 2008; Wolcott, 2008). For example, I noticed early on that the concept of sociomathematical norms was most useful in organizing and analyzing the data. I also noticed that teacher interviews were far more useful than student interviews. Hence, I stopped interviewing students and focused on classroom observations and teacher interviews to tease out the role of sociomathematical norms in Chen’s classroom. This focus resulted from careful analysis and re-analysis.

It is notable that there is an unavoidable element of researcher caprice involved in the focus on some themes at the cost of others. However, Wolcott (2008) notes that prolonged time in the field improves the trustworthiness of the themes the researcher does end up choosing. My semester-long stay in Chen’s classroom ensured that my selection of focus was not the result of simple caprice.
Wolcott (2010) wrote comfortingly that “there is nothing wrong with boundedness in a study; it is an essential quality that sets limits on what we can handle. The parameters of a study define its boundaries” (p. 36). This study was bound by the physical walls of the classroom, by the established number of participants (Chen and her 10 students), and the timeframe (one semester). However, it was also limited by the emerging theory which will emerge from the data. Bogdan and Biklen (2007) recommended waiting until the end of data collection to conduct the bulk of the data analysis. Conversely, Glaser and Strauss (2008) recommended that data analysis occur throughout the study beginning from the very first collection of data. I employed the technique called theoretical sampling outlined by Glaser and Strauss in light of Wolcott’s edict that the ethnographer exercise his or her mind as a delicate instrument of analysis. I began analyzing as soon as I began collecting data. However, I also continued to analyze and reanalyze the data for months after the data collection was completed.

Glaser and Strauss (2008) provided a clear outline of the process of theoretical sampling. This description echoed and also deepened Creswell’s (2013) description of data analysis. The researcher was to begin by collecting data in order to define local concepts and to make note of the relationships between these concepts. First, I observed the setting with the goal of describing the environment, the children, and the teacher. Next, I focused on describing the relationships between these. Over the course of the data collection, the researcher was to expand and minimize initial definitions of these local concepts as well as the relationships between them. For example, if the teacher engaged in a particularly interesting interaction with a specific child, I focused in on that
child in subsequent teacher interviews until I had fleshed out my concept of the child’s role and his or her relationship with the teacher. Because verbal interactions were the special focus of this study, I dedicated part of all of the interviews to clarifying with the teacher her perceptions of these verbal interactions.

The goal of clarifying local concepts was to move toward understanding their complex interactions as early as possible. As grounded theory emerged from the researcher’s perception of the data, it controlled the researcher’s subsequent analytic questions and thus the data he or she would collect. More specifically, as the researcher formed a working theory for the observed phenomenon, gaps in the theory would become apparent. These gaps would prompt the researcher to collect next data. There would be a degree of impersonality to this data collection and analysis because it emerged from theory. The groundedness of the theory would ensure its trustworthiness.

Trustworthiness, while not as absolute of the parallel quantitative term, validity, is the ideal in qualitative data analysis (Creswell, 2013). My own grounded theory emerged after the data collection was complete. Only after analyzing and re-analyzing the data did I discern that Chen’s efforts to change her practice could be understood in terms of developmentally appropriate practice.

While today’s ethnographer benefits from the work of a long line of workers in the tradition, particularly in the use of the tools they have designed through trial and error, there remains a persistently open-ended quality in the design which allows for what Wolcott (2008) terms mindwork. Wolcott (2010) contrasted quantitative research with qualitative research, writing:
Quantitative researchers have often confessed to “mining” their data, referring to going through data again and again to see if there is anything of interest that they might have missed in earlier efforts to comb through it. In a sense, qualitative researchers have engaged in similar practices. In my own data analysis process, I combed through my data set multiple times in order to discern themes and larger themes. However, my qualitative data analysis was also cumulative.

In the final estimation, Wolcott (2010) valued the cumulative understanding that the ethnographer gleaned from long immersion in the field. Like many other researchers (Bogdan & Biklen, 2007; Creswell, 2013), Wolcott urged the new researcher to bind his or her first study so as to learn the maximal amount from events that are occurred on the ground. I used the methodological steps outlined by Creswell as well as by Glaser & Strauss (2008), bearing in mind that no preformed set of steps could take the place of the ethnographic way of seeing which, Wolcott (2008) pointed out, was subjective.

Subjectivity thus shaped my data analysis. I used my subjective ways of seeing as a tool for limiting the study and also for gaining insight into our very human data. My professional, scholarly, and personal subjectivity yielded a uniquely useful glimpse into the possible lines of inquiry that others might later pursue in the area of mathematical conversations at the preschool level.

Researcher Bias

Becker (1998) wrote that we bring personal biases and personal theories as well as pet scientific theories to our work as qualitative researchers. In truth, we cannot arrive at the point of conducting a scholarly study without having read and internalized some
sociological ideas about conceptual relationships in social phenomena. We are bound to apply these ideas to our work consciously or unconsciously. Wolcott (2008), too, argued that our descriptions of data were always theory laden. What to do? It is not a question of ignoring these biases, but rather of being reflexive and confessional about them. Reflexivity permits us to be flexible in response to our data. In a word, we should be able to bracket our preconceived notions and be open to finding something completely new and surprising (Patton, 2002).

The ideal ethnographic study is that in which the ethnographer uncovers his or her own culture at the same time as he or she uncovers the culture of the participants. In fact, the finished product of the ethnographic study is the story of the participants’ culture as coauthored by both the researcher and the participants. Some believe the researcher necessarily weaves a part of his or her own story into the tale (Eisner & Peshkin, 1990). This was my stance at the beginning of the study. However, as I began to conduct interviews, I found my participating teacher was much more open and forthcoming with her views when I simply asked questions and listened than when I offered my own views of the observations I had made. In the end, I did not interweave my own story with hers as I had planned to do. She preferred to focus on her own experience and I was flexible in responding to this preference.

Patton (2002) suggested that it is beneficial for the researcher to focus on the participant’s story, setting aside our own, even if objectivity is never possible. Specifically, to the extent that our biases and predetermined ideas may get in the way of our listening to our participants with empathic neutrality, we must put them to
the side. I found that my own style of researching, at least in this setting was to let my participant, Chen, tell the story. I did not interweave my story with hers. I was more apt to bracket my thoughts and feelings and to capture Chen’s and her students’ voices in my final report.

I began by reflecting on my professional and scholarly biases which would come to bear most immediately upon the study. As a teacher and a scholar, I took a decidedly constructivist approach from the outset. The school in which I first taught as an assistant in a kindergarten teacher, on the Upper West Side of Manhattan, was politically liberal and pedagogically constructivist. The teachers included children in discussions about what rules to create for the classroom, the children voted on what activities they would engage in on a given day, and children were free to voice their thoughts and opinions to the teacher and to each other as we worked and played.

Realizing within a few weeks of working at this job, I made up my mind that I wanted to teach young children for the rest of my days, and I soon enrolled in a Master’s of Science degree in Early Childhood Education. Training in early childhood education in Manhattan in 2003 was, as a matter of course, politically liberal and pedagogically reform-based. The creation of that Master’s degree program was brand new, coming on the heels of overwhelming research that early childhood education was of much greater importance than previously recognized. The program was built upon the success of innovative curricula such as Montessori and Reggio Emilia. My professors taught me that children should spend most of their time in play and in high-quality conversation with teachers and with each other. Early childhood education should be equal parts
traditional, nurturing child care and cognitively stimulating academics--academics that were designed to be appropriate developmentally for the child’s age. Coming from a much more traditional early childhood education myself, I was inspired and convinced by what I was learning about this child-centered approach.

My professional biases are rooted in my own experiences as a child in school. I am both blessed and cursed with a very long memory. I still remember the days I spent in preschool when I was three and four years old. I remember moments I spent happily in play, and I also remember the chaotic moments in which we children pushed, hit, argued or otherwise played unfairly, mostly without interruption by our teacher. I look back on these memories regretting that our teachers did not structure our routines so that we might have gotten even more pleasure from our play. They could have varied our activities and scaffolded our unruly conversations, increasing our cognitive benefit from these activities and thus preventing our more negative behaviors. As a result of these early experiences and my later training in my Master’s degree program, I believe that there is an ideal balance between the child’s freedom and the teacher’s involvement.

I remember my elementary school in Doraville, Georgia, which I attended from kindergarten until seventh grade. It was a wonderful, orderly place in which children sat in rows and silently filled out worksheets and answered questions in one word responses only when called upon. I learned all the skills my teachers wanted me to--I was a dedicated student throughout my school career. However, my fondest memories of learning were those I spent at home writing short stories in my diary or composing little songs at the piano.
As I grew older, I looked back on my school years somewhat sadly, wondering why I did not have the freedom to enjoy my creative abilities at school. I wish I could have made the connection between my personal creativity and the collective discipline of the classroom. I believe if my teachers could have negotiated these two worlds, I could have gone further in both basic skills and in creative/critical thinking.

The ultimate conclusion I must draw from the above retrospection is that I am biased toward the research-based view that the classroom should be a balanced place in which the teacher socializes students into formal academic disciplines but also allows them to negotiate the personal understandings of subject matter with more formal understanding, thus constructing their own deep conceptual understandings. Personal involvement leads children to deep understanding of material. Deep understanding of subject matter, I believe, will allow students to use this knowledge critically and creatively.

When I entered the setting of my study, I was prone to look for this balance, and where I found it, I looked favorably upon it in my recordings of the data. I attempted to bracket this personal bias and to look a bit more impartially upon the data, recording it in a less personally biased than I might want to. It so happened that the participating teacher was also increasingly interested in designing instruction that was more in line with reform-based practice. However, she had to come to this conclusion on her own, without my interference.

While research on mathematics education has favored a constructivist approach (Cole & Wertsch, 1996), including the active use of teacher scaffolding,
mathematical conversations, researchers have also pointed out that there was a lack of consensus on what these practices should look like in the classroom. There was even less certainty about how preschool teachers should implement these research-based practices. Thus, it was critical for qualitative researchers in the area of early childhood mathematics education to make open-ended inquiries into the types of practices preschool teachers currently use so that we can next begin to evaluate their effectiveness.

As I have previously described, my experiences in both traditional classrooms and in more reform-oriented settings have biased me toward the reform-, constructivist-oriented approaches. This is the bias that I reflected on in my field journal along the way to ensure that I was bracketing appropriately. Grounded theory emerged out of my data in an impersonal way. I strived to make my participants’ voices heard. Giving voice to those who have seldom been heard from in the literature was an important goal of this study. The teacher and students involved were in the best position to direct future researchers by opening lines of inquiry in the area of mathematical conversations in the preschool classroom.

While Chen seemed like the ideal candidate for the study in the ways described above, problems of bias and conflict of interest are involved any time a researcher studies their own colleagues. Because I work in the same building with Chen, there was a danger that I would be biased in her favor because we are friends. Conversely, I may have been biased against them because I may estimate myself as a better teacher. The most powerful tool a qualitative researcher wields is reflexivity (Creswell, 2013). As I conducted the study, from the very beginning, I maintained a field journal in which I
painstakingly recorded my personal responses to the findings. Having identified this bias, I bracketed it, in the sense that Patton (2002) described. Writing about bias helped to bracket, or compartmentalize it, so that it did not unduly influence my understanding of the data.

The same can be said for conflicts of interest, which can threaten to shape the researcher’s perception of the findings. If I was too eager to protect my colleague, I may not have honestly report what I perceived to be condemning words or actions. Further, if I observed behavior on the part of the teacher that was inappropriate for the classroom, I may have been moved by loyalty to my colleague to conceal it. Of course, I was prepared to share with the school’s administration any inappropriate behavior on the part of my participating teacher. Beyond this responsibility, I simply wrote in my field journal about any conflict of interest that threatened to complicate my collection or analysis of data.

Ethical Issues

The current definition of ethics is only an iteration of a long historical tradition of ethics in qualitative research (Bogdan & Biklen, 2007). These ethics were shaped not only by the emerging form of qualitative research, but also by historical events that drew public attention to egregious violations of basic human rights. The Nazi experiments on prisoners in concentration camps during World War II led to the establishment of the Nuremberg Code, which outlined 10 ethical principles which should govern all scientific research involving human subjects (Hammersley & Traianou, 2012). The Tuskegee Syphilis Experiment, a 40-year study undertaken by the U.S. government to study the
growth and development of syphilis, included the deception of participants and the conscious failure to treat their treatable illness. This highly unethical biomedical experiment led to the creation of institutional review boards (IRBs) in all institutions that receive federal funding. The current study received approval from the Mercer University IRB in order to proceed.

The many historical events and agreements that have shaped a qualitative research tradition have put respect for the participants at the fore of the research process. Aside from the overarching directive to treat participants with respect, Bogdan and Biklen (2007) offer several specific and summarizing guidelines to ensure that researchers interact ethically with participants. They argued that participants should be fully informed as to what the research will require of them, including the time that will be spent in interviews. The researcher should be honest with participants and pursue their cooperation only as they are willing, making choices so as not to inconvenience them more than is necessary. Once the researcher has made an agreement with the participant or participants, he or she should not deviate from this agreement. The participants should be mindful of situations in which participants may feel coerced into participation and avoid these situations. The researcher should, at all times, ensure that the participants enjoy the freedom to quit participating. The researcher should also protect the participants’ privacy and, finally, report the findings truthfully.

Hammersley and Traianou (2012) have pointed out that while many emphasize the respect for and protection of participants, it is important also to underscore the need for research to be valid and useful to society. This is also a question of ethics. The utility
of the study should be worth the inconvenience to the participants. Ethical research is high-quality and answers important questions. Research comes at a fairly high price to participants in terms of the time they give, the effort they exert, and other forms of inconvenience, not to mention the risk of abuse or loss of privacy. Thus, the high potential costs of qualitative research are only outweighed when that research offers highly useful information to wider society.

This study was minimally intrusive and only involved one teacher and her 10 students. However, the potential benefits of the study were high. It offered a detailed glimpse into the mechanics of a preschool teacher’s mathematical conversations with students. No study of this kind has been undertaken previously. Thus, it was sorely needed in order to open up new lines of inquiry about the nature of mathematical conversations with young children.

Trustworthiness of the Study

For Hammersley and Traianou (2012), validity in qualitative research is often a question of clarity about what the data can and should teach the reader and what it cannot. Did the researcher make claims that the data cannot support? In a sense, qualitative reports are valid to the extent that they stretch local theory, but stop short of making untenable claims. The trustworthy researcher remains well aware of the fallibility of their evidence, so as not to go too far in interpreting its meaning. In the current study, I was fastidious in analyzing data simultaneously as I collected data in order to ensure that emerging research sub-questions truly emerged from the data (Glaser & Strauss, 2008). I also maintained memos and a fieldwork journal in order to engage in
self-reflection about biases that may have threatened to skew my interpretation of the data. In the beginning, I was very critical of Chen’s teacher-directed practice. I said nothing about my views during our first two interviews. It was Chen who noticed the discrepancy between her beliefs and her practices as she reviewed the transcripts. Theory emerged from the data in the context of this careful self-reflection on my part.

Both validity and protection for participants are built into the methods of sound qualitative research (Bogdan & Biklen, 2007). Because the aim of the method is to achieve authentic conversation with participants, participants possess a degree of control in determining the direction of inquiry as well as the methods of data collection. Initial research questions morph, and researchers may alter methods of collection based on participant input. The ethnographic method, in particular, can be traced back to the work of socially engaged researchers of the 1960’s for whom respect and empowerment of participants was paramount.

My special ethical concern was that my careful description of the setting and participants could make their identity apparent to readers. In addition, the ethical imperative to report findings honestly sometimes prompted me to describe teaching practices that were not considered best practices in the empirical literature. The worst-case scenario, then, was that I would inadvertently paint an unfavorable portrait of my participants and that others would recognize and disapprove of them. Bogdan and Biklen (2007) emphasized the importance of honestly reporting qualitative research findings. Not reporting miseducative teaching practices would amount to the perpetuation of inequity. The socially responsible goal of an education researcher is to
understand and report teaching practices be they effective, ineffective, or at worst, harmful. We must address them transparently so that these aspects of practice can be further examined and eventually improved for all students.

As an ethnographer, however, I took steps to understand all the classroom practices that I observed from the participant’s perspective. The teacher participant always had a chance to speak for herself in justification or defense of instructional practices. Also, through our conversations, it was made possible for the teacher to reflect upon and change the practices in question. Finally, I took measures to omit any descriptions that would clearly reveal the identities of those involved in the study. I balanced the goals of protecting privacy with that of offering detailed description of the context. The description of context was necessary for the reader to interpret the meaning of verbal exchanges in a trustworthy manner. These issues of balance were issues of both respect for participants and of research validity. A dedicated and thorough ethnographic researcher makes such decisions about balanced reporting as a matter of course.
CHAPTER 4

RESULTS AND ANALYSES

This chapter offers my research findings, focusing on how one preschool teacher and her 10 four- and five-year-old students overcame obstacles to mathematical conversation and co-constructed norms for speaking and reasoning mathematically. The overarching question was: What role do sociomathematical norms play in a preschool classroom? This initially generated two subquestions that guided this study:

a. How are sociomathematical norms constructed in a linguistically diverse classroom?

b. What conversation strategies does the teacher use to support the co-construction of sociomathematical norms?

Once research was underway, I revised the research questions slightly. The new overarching question was: What is the role of sociomathematical norms in a preschool classroom? Three subquestions developed:

a. How are sociomathematical norms constructed in a linguistically diverse classroom?

b. What are the obstacles to the co-construction of these norms?

c. What conversation strategies does the teacher use to support the co-construction of sociomathematical norms?
I added the second question when I observed that the teacher’s goal of incorporating conversation in her instruction was not an easy one. In fact, there were many obstacles to reaching this goal.

This chapter offers findings insofar as they answer the overarching research question as well as the three sub-questions. First, I describe a significant lesson in the participating teacher’s classroom. Next, I describe the conversation strategies the teacher used to support productive premath and mathematical conversation, especially in the whole group setting. Finally, I describe three distinct phases in the study. In each of these three phases, the participating teacher changed her practice so as to incorporate more student conversation in her lessons.

Conversation took place in three settings: whole group, small group, and pair work. The most important changes took place in the whole group setting although, paradoxically, the children conversed more freely in small group and pair work settings. As I describe these three phases, I describe obstacles to premath and mathematical conversations and the ways the teacher and children overcame these obstacles over time (see Appendix E for a comprehensive list of obstacles the teacher and children faced when trying to engage in conversation and Appendix F for a list of ways they overcame these obstacles).

Over the course of three phases, the teacher progressively developed her own unique developmentally appropriate conversation style with her students. Altogether, the findings of this study point to the importance of this teacher’s movement towards more child-centered conversation. I found that, as the teacher designed more developmentally
appropriate instruction, classroom conversations became more mathematically productive. The main negative cases that challenged this theory involved those students in the linguistic minority who remained on the sidelines of the conversation even to the end of the study. It is important to note that, as a trained preschool teacher myself, the epistemology I espoused was such that I was attuned to the emergence of these particular theoretical concepts. However, I provide ample evidence that these theoretical concepts describe the data in a robust manner. It is not known whether or not I would have conceived of this culminating theory without my prior reading about and working on developmentally appropriate practice during my own experience teaching preschool. This theoretical concept summarized the changes the teacher made to her practice and also made it clear when and why the teacher was effective in conversing with her students. As a theoretical concept, it was the best fit for explaining the data (Bazeley, 2013). It should be noted that this concept served as a local theory which was grounded in the particular data yielded by this particular sample. To form more substantive theory, the findings of this study should be compared to the conclusions of other, similar studies. In this chapter, it will be clear that I stretched the concept of developmental appropriateness to describe conversation strategies and their relationship to children’s resulting behavior. Developmentally appropriate practice is a well-researched, thoroughly-described concept in the literature (Copple & Bredekamp, 2010). This existing literature does not specifically describe the kinds of changes Chen made to her practice as being a part of developmentally appropriate practice. However, I will show how conversation strategies, for example, functioned in Chen’s classroom to support
developmentally appropriate conversation. They clearly rendered the conversations more accessible to her ten students.

The changes to the teacher’s conversation style led to greater opportunity for student conversation and thus for co-construction of sociomathematical norms. The example lesson below took place at the very beginning of the third phase of the study. It represented the most dramatic change in the teacher’s and children’s norms for interacting at that point in the study. It is clear in this example lesson that the teacher had developed strategies to support developmentally appropriate conversation.

An Overview: A Preschool Math Lesson

In the following lesson, Chen conversed with the children in a radically child-centered manner (Tzuo, 2007). This lesson was useful for analyzing concepts I will discuss throughout the chapter. It should be noted that Chen started out as more of an authoritarian teacher. She stated before the study commenced that children should converse in math lessons. Her practice, however, did not line up with her statement of intention. To my surprise, I did not observe math conversations occurring in her classroom in the beginning of the study. As time went on, however, Chen changed her practice to make it more child-centered and developmentally appropriate. She became a facilitator of conversation, especially in the whole group setting. She changed her practices as a result of her reflection on the transcripts of her lessons that I provided to her. She reflected on the transcripts. She also reflected on what she learned in her master’s degree program and from the speaker at a recent in-service. The following
lesson illustrated the dramatic changes that Chen made to her instruction as a result of these influences.

The study took place in a medium-sized private early childhood learning center in an Atlanta suburb. Although the administration provided a fairly traditional curriculum for the teachers to use, the teachers were not obligated to follow this curriculum to the letter. In a prekindergarten classroom in this school, Chen and Melanie were coteachers. Each was responsible for teaching half of the class during math, literacy, and science lessons. The children were divided into two groups for the teaching of these subjects. Melanie had ten children in her group and Chen had eleven children in her group. It should be noted that only ten of Chen’s students participated in this study.

On this particular day, as on any other day that I was present, Melanie took her small group outside to play at ten o’clock while Chen was to teach her small group. At eleven, they switched places and Melanie taught her small group. The teachers taught math every Thursday and some Wednesdays. However, the teachers taught math in incidental ways during many other times of every day. Further, they often placed math-related items in the various activity centers to promote mathematical conversations in those setting as well. The period during which I observed Chen’s group consisted of the last three months of the school year. Thus, Chen and the children had internalized teacher-centered practices through frequent repetition. This made the changes to their practices over the course of the study period that much more remarkable.

This particular Thursday morning, the children were moving around the room willy-nilly, talking to each other, looking at books, and playing with toys. Then, at the
usual time, Chen shook her tambourine. The children dropped what they were doing and came running to sit in the circle. Once in the circle, the children continued talking and playing together. Chen began to speak and some of the children began to listen. She told the class that she had noticed something worrisome in all their previous lessons. She said that she noticed the children always had a lot to say and not enough time to say it. “I know you get frustrated when you don’t get a chance to speak.” She told the children that today they were going to do nothing but talk, talk, and talk about the unit of the week, which was money. Now she had all the children’s attention. The children, Chen had observed in her review of the transcripts, wanted nothing more than to be free to talk. “Today, I want you to feel free to talk about money. You can ask questions and your friends can answer you. But should we all talk at once?” Chen explained that as long as the children took turns, they could speak without raising their hands.

In this particular lesson, Chen wanted to ask the most open-ended question possible to generate conversation. She asked the children, “What is money?”

A student named Emily answered, “Money is something that you buy.”

Chen turned to pose the question to the other students. She asked them if money was something you buy. Beth answered that actually money was not something you buy but rather something you give to people in stores to buy things. Chen did not tell Emily that she was wrong or Beth that she was right. Instead, she revoiced Beth’s answer for Emily and asked Emily what she thought about it. “So Emily, do you want to give your comment on that?” Chen asked. “She said that you’re wrong. So what do you think? You can tell her.”
Emily spoke directly to Beth. Speaking directly to a peer was a sociomathematical norm that the children began to co-construct starting with this lesson. “You were right. I didn’t know how to say that.” In this way, Beth helped clear up Emily’s misconception.

Further along in the conversation, Chen posed a question to the children. She asked them if they had ever had the experience of getting some money back in a store after paying some money. Logan spoke up and said that people who work in stores give you money back because they want you to have more money. Chen again revoiced his contribution to the group and asked the other children what they thought. A child named Hannah spoke directly to Logan and told him that he was wrong. She said that what Chen was talking about was called change.

Chen noticed that the two students in the linguistic minority were not contributing. Thus, she asked one of them, Aarav, if he thought this answer was correct. Aarav spoke Hindi at home and was often the quietest of the children during the lessons I observed. He answered to say that he did not know the answer. Other children gave incorrect answers in their turn. In this instance, Chen stepped into the role of the sharer of information. “Hold out ten fingers everybody.” The children all held out ten fingers. “Say we want to buy this globe right here and it’s five dollars. Each finger is a dollar. So take away five fingers. That’s how much you give the clerk at the store. How many do you have left?”

“Five dollars!” the children yelled out. Through further discussion with his peers, Logan’s misconception about change was cleared up and he understood why people gave
and received change. Starting with this lesson, the children began to co-construct the sociomathematical norm of clearing up one another’s misconceptions about premath and mathematical concepts.

This was one of Chen’s most successful whole group lessons in terms of her own goals of giving children more freedom in conversation. She used a variety of talk moves, including revoicing, recasting, and asking a child to respond to another child. She sought to include the students in the linguistic minority in conversation. Finally, she stated that she felt successful because she had facilitated a conversation among the children themselves. The children talked to Chen, but they also talked to each other.

In our subsequent interview, Chen stated she felt that she had facilitated an authentic conversation instead of simply imposing her views on the children. Chen shared that although she liked to step back whenever possible and let the children take the lead, it was still her job to step in when they were headed in an unproductive direction. In other words, she had an important role to play. She was the central participant in the classroom micro-community and it was her job to enculturate the children into the practice of productive premath and mathematical conversation.

In the previous scenario, I gave examples of strategies the teacher used to bring students into academic conversation, such as revoicing (Chapin & O’Connor, 2007) and recasting (Temple & Doerr, 2012). I gave examples of the sociomathematical norms (Lopez & Allal, 2007) the teacher and students had developed together to guide their conversations. The students knew they could speak directly to each other and without raising their hands as long as they did not become unruly. As stated above, Chen did not
start out the study with such a child-centered approach. In the next section, I describe the phases that Chen progressed through in order to design more developmentally appropriate practice. In the following section, I describe the conversation strategies that Chen developed to support productive premath and mathematical conversations. These conversation strategies, while not the only tools Chen made use of to this end, were the most important and useful strategies she developed to support this kind of conversation.

Teacher Conversation Strategies

The greatest change Chen made to her practice was to develop conversation strategies. The teacher’s conversation strategies played a role in moving the group from the initial verbal interactions phase to new rules for conversation phase and then to the sociomathematical norms phase of the study. In the next section, I focus on the various conversation strategies employed by the teacher.

Over the course of the study, the children were in conversation with herself and with each other, but Chen believed they were capable of more conversation if she were only able to employ the right techniques to support them. She saw in the transcripts that by and large she was still talking more than the children were. At this point in time, I pointed out to Chen that she was using two research-based conversation strategies. Specifically, I pointed out to her that she was the revoicing (Chapin & O’Connor, 2007) and recasting (Temple & Doerr, 2012). I was careful not to commend her or criticize her for her use of these strategies. I pointed them out to her using nonevaluative language. Chen was excited to learn that such research existed. She asked to see an example study. Consequently, I shared Chapin and O’Connor’s study with her. Chen carefully studied
this article and planned to incorporate all of the strategies outlined therein in her own classroom conversations.

In the weeks that followed, Chen began to use all of the conversation strategies described in that study. As time went on, Chen developed her own novel conversation strategies as well. Something “clicked” for Chen starting in the fifth week of the study so that she became skillful in using conversation strategies to orchestrate conversation between the children and herself, as well as among the children themselves. The conversation strategies were truly key in helping Chen to support more developmentally appropriate conversation. The strategies enabled Chen to balance student participation with her pedagogical agenda, although Chen did have to sacrifice much of this agenda in order to allow student voices to be heard. Faced with this quandary, Chen chose to sacrifice her teaching agenda to give children more freedom to converse and internalize the material in this way.

In this section, I describe in more detail both the interrogative and narrative conversation strategies that Chen employed during weeks five through twelve of the study. What the strategies had in common was that they helped extend conversation instead of stopping it, as Chen had been doing theretofore. The interrogative conversation strategies, or questioning strategies, included clarification requests, requests for a child to answer another child, requests for a child to add onto another child’s contribution, requests for a child to evaluate another child’s contribution, requests for a child to explain their reasoning, and finally requests for a child to repeat a peer’s utterance.
For the conversation strategies that were more narrative in nature, Chen offered answer choices when children had trouble answering a question, she revoiced student contributions, recast student contributions, and summarized conversations. A purpose that was common to all these conversation strategies was to help the students think explicitly about one another’s contributions so that they could respond to each other and keep the conversation moving productively. The more that the group extended and deepened conversation, the more likely it was that the group would co-construct sociomathematical norms. Thus, conversation strategies played a critical role in bringing children into more central participation (Lave & Wenger, 1999) in the classroom micro-community (Yackel & Cobb, 1996). Chen found that both interrogative and narrative conversation strategies were useful in supporting conversation and thus co-construction of sociomathematical norms. As I describe the conversation strategies below, I will also describe the shift they effected in the group from mere rule-following to the overt process of co-construction of sociomathematical norms.

Interrogative Conversation Strategies

Chen used at least six interrogative conversation strategies. Three of these were described by Chapin and O’Connor (2007) in their study on teacher talk moves. These were requests for a child to repeat another child’s contribution, requests for a student to explain his or her reasoning, and finally requests for a child to add onto another child’s contribution. Two of the interrogative conversation strategies were described in a study by Temple and Doerr (2012). Chen did not read this study. Rather, she developed these strategies on her own as a result of her ongoing reading of and reflection upon the
transcripts. As a pedagogical experimenter, Chen was capable of discovering possible conversation strategies on her own. These two strategies included requests for a child to clarify his or her answer or reasoning and requests for a child to evaluate another child’s contribution. The final conversation strategy was not described in previous literature. Rather it was a novel strategy that Chen developed. In this strategy, Chen requested that the child explain his or her reasoning for an answer.

Clarification request. One of the simpler conversation strategies was the clarification request. It was not one of the strategies that Chen used most often. However, it was an important strategy in that it allowed Chen and the children to extend conversation and to thus co-construct sociomathematical norms together. Chen tried whenever possible to elicit clarifications from the children. Chen requested children to offer clarifications when their answers seemed incomplete or incomprehensible. When the children were not forthcoming with clarifications of their answers, Chen supplied the clarification herself.

In the fifth week, Chen asked the children what they used to tell time at home. Emily stated that she saw the time on her mother’s phone. Chen asked for clarification, asking Emily if her mother’s phone showed all the numbers like their clock did or if it just showed the time. Emily answered erroneously that her mother’s phone showed all of the numbers. Chen said that she did not think this was the case, because phones showed the time digitally. Chen went to her purse to get out her own phone. Chen called the children’s attention to the difference between their classroom clock, which was analog, and her phone, which was digital. Emily nodded her head, accepting that her answer had
been incorrect. Thus, Chen used this conversation strategy to help co-construct the sociomathematical norm of talking through disagreements.

Request for a child to answer another child. A conversation strategy that Chen used often was the request for a child to answer another child. Chen explained that she used this strategy frequently to enculturate the children into the practice of speaking directly to one another, and into taking more responsibility for the conversation. This strategy was part of an ongoing pedagogical experiment. Chen did not just ask the children to respond to each other when their answers were incorrect, but also when their answers were correct. When a contribution was incorrect, Chen called on students to answer each other so that they could correct the child who had given the incorrect contribution. But she also asked children to answer each other when their answers were correct in order to showcase this correct answer. She also did so to extend and deepen the first child’s idea. In the following case, Chen asked the children to answer the first child to correct her mistake.

In the fifth week, during the conversation about money, the children spoke for a while about how you get money when you have none. Hannah held a mistaken notion that people had to pay to get a job. Chen asked Hannah how you get the money to pay for the job. Other children protested that Hannah’s contribution was not correct. At the sound of these protests, Hannah asked, then how do you get money? Chen asked the children to answer Hannah’s question. This instance is a good example of how Chen’s students had begun to turn to each other for answers. At an earlier phase, Hannah may have insisted that her answer was correct. However, now, she had internalized the norm
of receiving information from her peers. She asked her friends for information about how you get money when you have none. Olivia finally spoke up and said that her mother got money by working at her job. She added that her mother got the job without paying for it. Chen asked Olivia to answer Hannah directly.

“Hannah, you wanted to know how people get money. Listen to what Olivia has to say.”

Olivia turned directly to Hannah to say, “You get money by working at a job. That’s what my mommy does.”

As illustrated by this example, the conversation strategy of requesting children to answer each other prompted them to converse directly with each other and not just with the teacher. In this example, Chen still had to prompt the children to answer each other’s questions. However, the children were quick to co-construct this norm with the teacher. Later in that same conversation, Hannah asked how grown-ups obtain jobs if they do not have to pay for them. Mahesh answered her directly without being called on. With the help of the conversation strategy, he had internalized the norm of speaking directly to his peer to share information. Mahesh told Hannah that to get a job you have to dress up in nice clothes, go to the place, and ask them if you can work there. He said that he knew this because his dad did that to get a job. Over the course of the study, the children answered one another’s questions with ever increasing frequency, evidence that Chen’s conversation strategy successfully enculturated them into this practice. As the weeks rolled by, the children increasingly asked each other questions directly, and their peers answered them directly. Chen stood by to scaffold the conversation where appropriate.
However, the children took on more and more responsibility as the weeks passed and they gained practice conversing with each other.

Request for a child to add on to a peer’s contribution. A conversation strategy Chen used often was to ask a child to add on to what another child had said. This conversation strategy, like several others, served the purpose of extending the conversation and also of deepening the conversation to examine the first child’s contribution. Conversation strategies helped students make sense of one another’s contributions. By asking a child to add on to a peer’s contribution, Chen made it clear that the first student’s utterance was valid, yet incomplete, and would benefit from some expansion.

In week nine, during an open-ended conversation that Chen orchestrated before putting the children in pairs for math games, Chen asked the children what the words addition and subtraction meant. Mahesh said that in subtraction, you use a minus. Chen praised him for his contribution and asked Logan if he had something to add to Mahesh’s example. Logan said that in subtraction, you count down. Chen agree with him and then called on Aarav, asking him to add on to what Mahesh and Logan had said. This conversation took place in the whole group setting. Students in the linguistic minority spoke less in this setting. Perhaps this is why Aarav stumbled a bit over his words when he answered. He said, “Um, a little bit.”

Chen asked him, “What do you mean by a little bit?”

Aarav answered, “You will have just a little bit.”
Chen guessed at his meaning, saying, “So when you minus something, you go from a lot to a little bit? Is that what you’re saying?”

Aarav nodded his head.

This example from the transcripts demonstrates that by requesting children to add onto one another’s contributions, Chen engaged more of the children in conversation, which was the overarching purpose of the co-construction of sociomathematical norms. This conversation strategy also served to signal to the children that they should be paying attention to one another’s contributions in case the teacher might ask them to add something on. Chen knew this attentiveness to one another’s contributions made it more likely that the children would converse with each other.

Request for a child to evaluate a peer’s contribution. The conversation strategy of requesting that a child evaluate the contribution of another child is one that Chen used from the very beginning of the study. Evaluation requests cued the children to pay attention to and think critically about one another’s contributions. Chen’s habitual use of evaluation requests in the third phase of the study also encouraged the children to be ready to verbally evaluate one another’s contributions. In lesson five, Chen opened the lesson by asking the children what money was. Emily answered that money was something that you buy. Chen asked Beth to evaluate Emily’s answer by asking her if it was correct. Beth corrected Emily saying that money is something you take with you to the store to buy things. Emily received this correction with appreciation, saying that she knew Beth was right and that she just did not know how to say it. In this case, the use of
this conversation strategy led to the children talking directly to each other, talking through a disagreement, and clearing up one another’s misconception.

Request to repeat a peer’s utterance. Chen used this conversation strategy several times, especially when a student gave a contribution that she wanted the other children to internalize or respond to. Often, the simple repetition of a child’s utterance led to further productive conversation. In week five, when the teacher and children were discussing change, Hannah was one of the children who understood why and how people gave and received change at stores. Not all of the children grasped this concept, but Hannah both understood the concept and was able to articulate her understanding clearly. Hannah explained that when you give a person at a store too much money, they give you back the extra. That is called change, Hannah said.

When Chen asked Elodie what she thought change was, Elodie said she did not know. Chen asked her to repeat Hannah’s contribution. Elodie repeated Hannah’s statement that people at the store had to give you extra money. Elodie added that that had happened to her when she was with her grandparents at a restaurant. Chen asked the children to raise their hands if they also had personal experience with the process Hannah and Elodie had described. Most of the children raised their hands. Here and on many other occasions in the third phase of the study, this conversation strategy helped Chen to engage Elodie in conversation and also to elicit her personal connection to the topic. In so doing, Chen ensured that Elodie and the other children listening would be more likely to internalize the material as well as the norm of listening carefully to one another’s speech so as to respond to it.
Request for a child to explain reasoning. Chen often used this conversation strategy in phase three of the study. Chen was not simply interested in the correct answer to the problems she posed to the children; she wanted to know what reasoning they used to arrive at the answer. In lesson six, she used this strategy to elicit Nancy’s reasoning while she was engaged in matching coins to circles on a game sheet. Nancy had matched all the coins to the circles on the sheet and had requested that Chen check her work. Chen was interested in the reasoning that Nancy had used to solve these problems. Thus, she asked Nancy how she had matched the coins to the correct circles. Chen specified that she wanted to know how Nancy thought through the problem: “I want to know how you reasoned. That means I want to know what you were thinking when you placed these coins on these circles.”

Nancy answered that she had looked at the size of the circle. Chen asked her if that was how she solved all the problems. Nancy answered that she also looked at the numbers. Chen gave Nancy a high five and thanked her for explaining her reasoning. On this and other occasions, Chen used the conversation strategy of requesting a child to explain his or her reasoning to enculturate the child into the practice of verbalizing her reasoning. Chen wanted the children to know that it was not enough to answer a problem correctly. This strategy ensured that the child would think conceptually about the question he or she had answered so as to be able to communicate her reasoning to another person. In conjunction with the interrogative strategies, Chen used narrative strategies in which she described student contributions in various ways. I turn to describe these narrative strategies in the next section.
Narrative Conversation Strategies

Chen used at least four narrative conversation strategies. One strategy, that of revoicing a student’s contribution, Chen learned about from the Chapin and O’Connor’s (2007) study. Chen used this strategy constantly. Chen also recast student contributions very often. This strategy was described in Temple and Doerr’s (2012) study, but Chen devised this strategy without having read this study. The other two narrative conversation strategies, Chen developed as a result of reflecting upon her practice and conducting pedagogical experiments. These were not described in the literature I had found. These two strategies included offering students possible answer choices when they were struggling to answer a question, and summarizing the gist of a conversation while it was underway.

Teacher revoicing student contribution. According to Chapin and O’Connor (2007), teachers revoiced a student contribution for two purposes. First, the teacher would repeat a student’s words to validate his or her contribution. Second, the teacher would use it to encourage the rest of the group to consider and respond to the child’s contribution. Chen used this strategy from the beginning of the study even before she read Chapin and O’Connor’s study. At first, she simply used the strategy to signal to the children that she wanted them to stick to the topic at hand. However, after reading Chapin and O’Connor’s work, she used the strategy for the purposes described in that literature.

In week five, during the productive conversation about money, Chen asked the children what money was. Emily answered that money was something that you buy.
Chen revoiced Emily’s contribution. She did so on this and on other occasions to offer Emily’s contribution for the children’s explicit consideration so that they could respond to it. Beth responded in disagreement. Beth stated that money was not something that you buy, but rather something you give people in stores to buy things. Chen revoiced this response and presented it explicitly to Emily. Then, Emily spoke directly to Beth that she was right and that she just did not know how to say what Beth had said. On this and other occasions, this strategy assisted the children in communicating directly with each other. It also helped the children to talk through a disagreement amicably.

Recasting a student’s contribution. Recasting was another conversation strategy that Chen used from the very beginning of the study. Recasting was a conversation strategy described in a study by Temple and Doerr (2012). Chen was not exposed to this study. Rather, Chen developed this strategy through her own considerable efforts to engage children in more productive conversation. This strategy was similar to revoicing in that the teacher repeated the child’s contribution in part. However, in recasting, the teacher altered or added onto the child’s contribution so as to present the answer in a more technically correct form. Chen used this strategy often. At first, she used it to focus the children on a single subject. As time went on and Chen came to respect her students’ contributions more, she used it to encourage the group to consider the contribution explicitly and respond to it.

In the first week of the study, during the lesson on the teen numbers, Chen talked about the number eleven. She showed the children the number 11 as it was written on a
card Chen had prepared. She asked the children, “How do we get to number twelve from magic ten?”

Olivia answered, “By counting to ten and then adding some.”

Chen recast Olivia’s answer by saying, “By counting to ten and then adding two.”

Chen stated in an interview that she felt that by recasting Olivia’s answer instead of telling her she was wrong, she was validating Olivia’s effort while still offering the children correct information. As the study progressed, Chen availed herself of this strategy to prompt children to respond to each other, thus moving conversation forward.

Offering answer choices. Less often, Chen offered the children possible answers to choose from when she was aware that the question she was asking was difficult for them. For example, during a lesson on time, Chen asked the children what they did at each hour of the day. Few of the children had strong enough a grasp of the concept of time to answer accurately. Thus, Chen offered some possible answer. For example, when she asked the children what they did at four o’clock in the afternoon, she offered some possible answers to this question. First, she reminded them that they get up from nap right before four o’clock. Then, she asked, “So...right after nap, what do we do? Do we have math? Do we read books? Do we play outside? Or do we have snack?”

Most of the children answered in chorus, “We have snack!”

Chen praised them for giving the correct answer. As illustrated by this example, Chen used this strategy when she wanted to hand over responsibility for the answer to the children, but she was aware that they needed assistance.
Summarizing conversation. A final conversation strategy that Chen used occasionally was to summarize the conversation that had taken place up until a certain point. For example, when she put a question to the children and then they answered with many disparate but interesting ideas, Chen stopped the children from answering momentarily to summarize all of the answers the children had given. Chen did so to give the children the opportunity to think about all of these ideas and then to address each one without getting confused by the multiplicity of answers.

In one premath conversation, Chen used the strategy of summarizing the conversation before moving forward to address each point. In this conversation about the bake sale and money, she asked the children what people do with money when they earn it. Hannah said that they spend it. Elodie said they could give it to someone as a present. Logan said they could save it. Chen said, 

Okay, wait just a second before we continue. I’m hearing many different answers and they’re all good ones. Someone said they can spend their money. Someone said they could give it away as a gift. And a third person said they could save it. All of these are excellent examples of what people can do with money when they earn it. Let’s talk about spending money first. . . .

The group went on to talk about different ways people can spend money. Only after discussing this topic at some length did they go on to talk about the other things people could do with money.

This conversation strategy, which Chen used in the third phase of the study, can be contrasted with Chen’s speech behaviors in the first phase. In the first phase of the
study, Chen stopped the children from talking in order to correct them or to deliver other information or instructions. With the conversation strategy just described, Chen stopped the children from talking, but she did so in order to extend their conversation. If Chen had simply allowed the children to yell out more answers, it is unlikely that the children would have gone on to address each idea. However, with this intervention, the children were enabled to thoroughly explore each of the ideas they themselves had proposed. Like the other conversation strategies, this one was intended to deepen and extend conversation.

The use of the above conversation strategies helped Chen overcome her fear of giving children more responsibility in premath and mathematical conversations. She continued this struggle with her fears even to the study’s end. It was clear that she would continue to negotiate with her students in the future when it came to how much freedom she gave them in conversations. However, Chen may never overcome all of her fears of giving the children responsibility for conversation. Perhaps Chen’s fears were grounded in reality. It appeared that Chen was right in withholding some responsibility from the children. While it was developmentally appropriate to give the children more responsibility, it was not appropriate to give them all the responsibility before they were ready (Vygotsky, 1934/1978). Her role as preschool teacher meant that she had an important role to play in facilitating and orchestrating conversation. This was where conversation strategies came into play.

In the next three sections, I discuss the phases Chen passed through over the course of the study. In so doing, I will discuss the ways that conversation strategies and
other strategies enabled Chen to overcome obstacles to conversation. In the next section, I chart the process by which Chen amended her practice, overcoming obstacles through bravery and determination. This process took place across the three phases of the study: weeks one and two, weeks three and four, and weeks five through twelve.

The Three Phases

In the following sections, I describe the three phases over the course of which Chen changed her practice. I reveal how these changes gave children opportunities to co-construct sociomathematical norms with Chen and with each other. Over the course of these three distinct phases, Chen changed her practice to make it more conversation-rich. I describe the obstacles that Chen had to overcome in fostering conversation. I also demonstrate that Chen had to support developmentally appropriate conversation in order for the children to co-construct sociomathematical norms.

The first phase I call initial verbal interactions. I use the word interactions because Chen was not yet engaging children in true conversations at this phase. The second phase I call new rules for conversation. Unlike in the first phase, Chen was now consciously developing rules in this phase for how true conversations should unfold during her math lessons. I use the word rules instead of norms to describe this phase because the children had not yet truly adopted the rules so that they would become norms. Chen was imposing rules upon the children in this phase. I call the third and final phase sociomathematical norms. In this phase, the children had truly adopted the rules and made them their own. Thus, in this phase, I speak of norms instead of rules.
Although in reality, every social situation is guided by co-constructed norms, this co-construction process did not occur overtly until the third and final phase. By the end of this last phase, Chen had authored her own child-centered, developmentally appropriate conversational style that enabled her to include all of the children, at least to some extent. It is important to note that these rules were in effect most of all during whole group meetings. In small group and pair work times, the children were allowed to speak more freely even from the beginning. However, their conversations in these settings were not as mathematically productive in the first phase as they were in the next two phases.

The concept of developmental appropriateness was the best fit for explaining the data on a theoretical level (Bazeley, 2013). When Chen changed her practices to make them more developmentally appropriate, conversations with the children were noticeably more productive. When her practice was not developmentally appropriate, conversation was not productive and the children went off-task and acted out. The concept of developmental appropriateness was thus useful in describing the teacher’s effective practice and the children’s resulting conversation.

The data showed that, at the start of the study, Chen was an unprepared pedagogue. Although she had completed a master’s degree in early childhood education and had recently attended an in-service about academic conversations at the preschool level, Chen had received no explicit training in how to engage children in academic conversations—or more specifically, in mathematical conversations. This was the reason
Chen had to proceed through phases in order to achieve her goals for productive classroom conversation.

Chen may have been unprepared, but she was eager to learn. She entered the study with the goal of improving her practice. Chen changed her practice over the course of the three phases with the help of some scholarly literature, through reflection on the transcripts I shared with her, and most of all through her own pedagogical experimentation. The most change occurred in the context of whole group lessons, although the changes in this setting spilled over into the small group and pair work settings. Chen had to overcome many personal fears in order to begin changing her mathematical instruction practice to incorporate more conversation. In her own words, she had to muster bravery to change her practice. Bravery was a requirement because she was afraid that her administrators would disapprove, she was afraid that the parents would complain if she deviated from the traditional style of teaching math, and finally, and she was afraid that the children might not learn this way in spite of what she had read about the effectiveness of child-centered math teaching practice.

In interviews, Chen said she felt that the culture of the school was not hospitable to this type of reform-based practice. Over the course of this study, Chen her fears enough to radically change her practice. However, she never completely overcame the worry that she might somehow get into trouble for implementing these reform-based practices. This continued worry was a reflection on the setting in which she taught. It was clear that mixed messages were sent to the teachers about the style of teaching that they should adopt.
I pondered why Chen was only now willing to overcome these fears to engage in such radical experimentation. Through observations and interviews, I gathered that part of the reason she had asked to participate was because of the prestige of participating in my doctoral level study. It was improbable that Chen would have changed her practice to the extent that she did were it not for her participation in my study. In this sense, the influence my presence had on Chen’s practice was a limitation to the ethnographic nature of the study. I cannot be sure how much Chen would have changed her practice had I not been present.

Chen expressed that she felt insulated from outside disapproval because she was participating in an official study. It should be noted, however, that the radical changes to her practice were all generated by Chen. She was the one who took the initiative and made progress towards her goals as a result. I was careful not to influence Chen in any further capacity. Chen confessed that she had been unhappy with how she had been teaching math prior to this study. She had been hoping for impetus to change her practice. As a result of the changes she made to her practice, both she and her students appeared far happier and more engaged. Chen permitted students to participate for more, and as a result, they remembered more of what they learned. Chen’s practice, by the end of the study, was clearly developmentally appropriate. It enabled children of four and five years old to participate in conversation, whereas Chen’s previous strategies had not enabled to do so to the same degree.

Observations and interviews allowed me to ascertain how this preschool teacher was capable of drawing upon her own resources to overcome many obstacles to engage
children in premath and mathematical conversations. The final picture that emerged was that of a teacher who took the initiative to devise her own partially successful style of mathematical conversation. This picture included that of students who fell easily into step with these changes. The study told the story of an emerging preschool-level discourse community. Chen was proud of the changes she made to her practice. She used the words “brave” and “bravery” when she spoke of the initiative she was taking in revising her instructional practices. Although she ultimately had no mentor or text to refer to in order to feel certain she was doing the right thing, she felt proud of her strenuous efforts to improve. She felt certain she was moving in the right direction over the course of this study. Chen surmised from the transcripts, however, that the changes she was making did not benefit all of the students equally. Table 1 provides information about the home language of the students who participated in the study. Those children whose home language was not English participated the least in premath and mathematical conversations even to the end of the study.
Table 1

*Children in Chen’s Class with Demographic Information*

<table>
<thead>
<tr>
<th>Name of Child (Pseudonym)</th>
<th>Gender</th>
<th>Age</th>
<th>Language Spoken at Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aarav</td>
<td>Male</td>
<td>5 years old</td>
<td>Hindi</td>
</tr>
<tr>
<td>Beth</td>
<td>Female</td>
<td>4.5 years old</td>
<td>English</td>
</tr>
<tr>
<td>Elodie</td>
<td>Female</td>
<td>5 years old</td>
<td>English</td>
</tr>
<tr>
<td>Emily</td>
<td>Female</td>
<td>4.5 years old</td>
<td>English</td>
</tr>
<tr>
<td>Forrest</td>
<td>Male</td>
<td>4.5 years old</td>
<td>Mandarin</td>
</tr>
<tr>
<td>Hannah</td>
<td>Female</td>
<td>5 years old</td>
<td>English</td>
</tr>
<tr>
<td>Logan</td>
<td>Male</td>
<td>4.5 years old</td>
<td>English</td>
</tr>
<tr>
<td>Nancy</td>
<td>Female</td>
<td>4.5 years old</td>
<td>English</td>
</tr>
<tr>
<td>Olivia</td>
<td>Female</td>
<td>5 years old</td>
<td>English</td>
</tr>
</tbody>
</table>

Initial Verbal Interactions: Weeks One and Two

Careful review of the data revealed that Chen and her students progressed through three phases over the course of the study. These changes started with Chen. Chen received her master’s degree from a university in Georgia. In this master’s degree program, Chen learned that she should engage children in conversation. However, once she began working as a teacher in her present school, she conformed to the culture there, which encouraged the use of a teacher-directed style. In interviews, Chen shared with me that she always had it in the back of her mind that she should encourage the children to speak more. However, she faced obstacles in accomplishing this goal. Obstacles were
presented by the teacher and by the students alike. In this section, as I describe Chen’s initial practice, I will also describe the challenges Chen faced as she attempted to engage children in verbal interactions.

Teacher-Centric Rules

As an unprepared pedagogue, Chen had no way of knowing if a discourse-rich style of instruction would truly work for her students and all the other stakeholders involved. She had never tried teaching in this way, and she had never seen it in action. It’s important to note that when we discussed Chen’s discourse practices, we mainly referred to the lessons in the whole group setting. This setting was the most important one for Chen because, as she stated, she believed this was where the most premath and math learning took place. It is also notable that the children conversed more in the small group and pair work settings. I noticed that as Chen changed her practices in the whole group setting, children’s conversation in the other settings was enriched as well.

Chen’s habitual teacher-centered style of teaching was interrupted when the administration brought in a speaker for an in-service one month before the study began.

Chen had already agreed to participate in my study, which I had described to her as being about mathematical conversations in a preschool classroom. Coincidentally, this speaker encouraged the teachers to engage their students in rich conversation in all learning domains. This speaker encouraged the staff to “strive for five” or to go back and forth with a child five times—going beyond the usual routine of asking a question, listening to the child’s response, and then evaluating that response. This routine is known as the initiate-response-evaluate (IRE) model (Mehan, 1979). The speaker emphasized
the importance of academic language development for preschool-age children. Chen was inspired by the speaker’s encouragement to involve students in conversation rather than just forcing them to listen passively. This speaker’s message reinforced what Chen had learned in her reform-minded master’s degree program. Thus, Chen may have been bound to change her practice to some extent even without my presence in her classroom.

In the first interview that I conducted with Chen before I began to observe her, Chen stated her desire to converse with the students during math lessons. However, she also expressed misgivings about how to go about accomplishing this goal. She believed that she possessed some basic skills in supporting premathematical and mathematical conversations with her students. Although Chen was not aware of the rules that she had put in place for verbal interactions during these first two weeks, what emerged from the data was that there were a number of hard and fast rules that governed verbal interactions during this first two week phase of the study. Contrary to how Chen saw her own practice, the rules she upheld were so rigid that conversation was not possible. For this reason, I refer to the first two-week phase as consisting of initial verbal interactions, as opposed to conversations. Chen faced numerous obstacles to engaging her students in conversation. Some of these obstacles originated with her, while others originated with her students.

As I observed Chen in the first two weeks of the study, I noticed that she taught in a very teacher-directed style (Tzuo, 2007). I observed that she delivered information and instructions. She often spoke for extended times, but she never allowed the children to speak at length. It was her practice to cue the children to finish her sentence or even her
word. Finally, I observed her to cue the children when to answer in chorus. Otherwise, she never allowed them to speak more than one person at a time. As for the children, they had to raise their hands and wait to be called on. When called on, they were expected to answer specific questions only and their answers had to be short. Personal connections to lesson content were discouraged. Chen gave children little opportunity to make their voices heard. Chen’s practice during this period was not developmentally appropriate.

As I reviewed the data, it became clear that the teacher had to discipline the children the most during this phase. In the first two weeks of the study, Chen had to discipline her students 38 times. Another characteristic of this phase was that children often became frustrated and/or acted out when the teacher stopped them from speaking. In this first phase, the children’s off-task behavior posed an obstacle to verbal interactions. Finally, it emerged that the two students in the linguistic minority were largely excluded from the conversation. These were the rules and patterns, so to speak, that characterized the initial verbal interactions that took place during data. I found that clear rules governed these interactions, especially during the whole group lessons.

Some of the verbal interactions I subsequently describe were premathematical, in that they established the context in which mathematical conversation could take place. Other interactions were truly mathematical in nature. In either case, I found Chen to engage in unconscious rule construction.

The data demonstrated that Chen enforced three teacher-centric rules. The first was that only the teacher was to deliver information and instructions. The second was
that only the teacher could speak at length. The third was that the teacher could prompt
the children to finish her utterances. The fourth and final rule was that the teacher could
prompt the children to answer in chorus. I describe the child-centric rules in a subsequent
section. All of these rules were exemplary of the kind of rigid rules for communication
Chen imposed upon the children in the first two weeks. These rules emerged out of
certain fears Chen had that prevented her from engaging her students in conversation.
She feared that she would not get through her pedagogical agenda. She was afraid of
children’s off-topic answers because they might derail the conversation. She was also
afraid of teaching differently than the other teachers in the school.

Only the teacher could deliver information and instructions. One rule that emerged was that the teacher was in charge of delivering information and instructions. In the first two weeks, I observed that delivering information and delivering instruction were Chen’s primary modes of expression. In the second week, during a lesson on time, Chen read a book called *What Time is it?* by Bobby Caulman. Chen read this book mainly to deliver information to the children and not to engage them in conversation. The children, for their parts, kept their hands up for most of the lesson hoping that Chen would call on them so they could share. With few exceptions, Chen did not call on the children while she read this book. She did not give them the opportunity to converse at the top of their ZPD (Vygotsky, 1934/1978).

Chen read the book and shared with the children that the clock tells people what
time it was, but the calendar tells them what day it was. She shared that there were
longer periods of time than hours and days. These longer periods, she said, were called
seasons. There were four seasons, she shared. There were also even longer periods in history. Chen showed a picture of dinosaurs and said that they lived a long time before the children were born. After Chen shared each of these facts, the children raised their hands to share thoughts and questions. Chen only called on children to answer her specific questions. She did not allow any of the children to share creatively. Her goal for the lesson was simply to share as much information as possible.

Chen also gave instructions. She asked children to turn pages or to bring props to the circle to help her teach the lesson. By focusing solely on delivering information and instructions, Chen missed out on many opportunities to engage children in conversation. When the children protested, Chen told them they could share after she finished the book. The children’s protests took up time that could have been used for them to share after the book was finished. As was typical of this period, Chen found that after she was done reading the book, she was short on time. Therefore, she did not give the children a chance to share their ideas as she had promised she would.

During the first two weeks of the study, Chen stuck to her pedagogical agenda and did not sacrifice any of it to permit for free conversation. The lessons were completely teacher-directed and not developmentally appropriate. In reviewing the transcripts, Chen took note that the children heartily protested the restrictions she had placed on them. The children were not happy acting merely as recipients of information and instructions. Chen also acknowledged the many opportunities she had missed that would have allowed her to draw out conversation. Chen’s reflection on these facts helped her change her practice in the next phase of the study.
Teacher prompted children to finish her utterances. Another speech habit of Chen’s was to prompt children to finish her sentences or even her words. This was another speech behavior that Chen engaged in throughout the first two weeks. As described above, Chen often delivered information to students. Each time she finished delivering information, she asked the children sequences of closed-ended questions. Most of these closed-ended questions took the form of prompts to finish her sentence or even her word. These closed-ended questions left no opportunity for the children to answer creatively or with personal flourish. In the second week of the study, when Chen reviewed the teen numbers, Chen started by pointing to each of the teen numbers on the hundred chart and describing what they looked like. After having provided this description of numbers eleven through nineteen, she went back to the number eleven and said, “The number 11 is a 10 and a . . . what, Elodie?” She called on Elodie because she was raising her hand.

Elodie answered incorrectly. She finished Chen’s utterance by saying, “Magic ten.”

Chen called on Logan next because he was raising his hand. Once again, she prompted him to finish her sentence: “Logan, the number 11 is a magic 10 and a . . .”

Logan answered, “Magic eleven.”

Chen responded to him by saying that we only talked about the number 10 being magic. The number 11, she said, was not magic. The children’s lack of understanding of the content presented another obstacle to productive verbal interaction.
After receiving two wrong answers, Chen provided clarification by pointing out that the first numeral in each teen number represented the magic ten. I gathered that Chen’s rationale in prompting children to finish her sentences during the first two weeks was that it would keep them engaged in the lesson. However, I observed that the strategy was ineffective for that purpose. In this first phase of the study, Chen lacked useful strategies to engage the children in the lesson. She was an unprepared pedagogue.

Teacher prompted children to answer in chorus. A third speech behavior I observed Chen to engage in was to prompt the children to answer in chorus. This strategy was also exemplary of Chen’s rigid communication style in the first two weeks of the study. Chen engaged in this speech behavior more in the first two lessons (initial verbal interaction phase) than she did in subsequent phases (new rules for conversation phase and sociomathematical norms phase). Typically, during the first two weeks, only one child was allowed to speak at a time. However, when Chen cued the children, they knew to answer in chorus. In other words, the children were allowed to speak at the same time when they were cued to do so by Chen. This was a rule that was obviously already in place before the beginning of the study. This rule, like the one described above, also prevented children from expressing themselves freely. They had to answer in a prescribed manner. This rule was, therefore, not developmentally appropriate.

As described above, Chen asked Nancy to make the number 11 out of Unifix cubes. She instructed Nancy to make a 10 bar first. After Nancy had connected 10 cubes together, Chen held it up for all the children to see, and said, “Thank you, Nancy. Everybody, now we have made a magic . . .”
“Ten!” answered the children in chorus.

As in this instance, Chen typically cued the children to answer in chorus for questions she believed they all knew the answer to. She cued the group to speak together by saying, “Everybody”. In interviews, Chen stated that she felt these demonstrations gave the children confidence. However, there were few instances during the lessons in this phase when the children gave correct answers. Thus, once again, one of the rules that governed Chen’s behavior was not as helpful as she believed it to be. Although Chen made it clear she only wanted the children to answer her specific questions and to answer in chorus, the children persisted in expressing themselves in other ways. Chen did not know what to do with these free expressions. Reflecting on the transcripts later, Chen was able to see that her practices did not lead to productive conversation. This recognition led her to think about more productive ways to engage the children. In addition to the teacher-centric rules, expectations about the children’s behavior surfaced during the first two weeks of the study. I turn to these child-centric rules in the next section.

Child-Centric Rules

Just as certain rules governed Chen’s speech behaviors, Chen also enforced rules to govern the types of speech behaviors children could engage in. Once again, Chen did not express awareness of these rules. As I reviewed the data, it became clear that children were not allowed to speak at length, they had to raise their hands and wait to speak, they were only allowed to give short answers, and finally they were discouraged
from making personal connections to the lesson content. These were the rules that
governed the children’s speech during the first two weeks of the study.

I observed that the children felt uncomfortably restricted by these rules. They
frequently broke the rules and acted out in response to the rules. The rigidity of these
rules, like the teacher-centric rules, gave the children little room to contribute in the way
they would have liked to. As a result, they were unengaged or rebellious in their
behavior. Chen took note of these behaviors as she reviewed the transcripts.
Paradoxically, the children’s persistence in acting out and breaking these rules
strengthened Chen’s resolve to change her practice. The children’s negative behavior
inspired her to design instruction that was a better fit for how her students learned.

Children were not allowed to speak at length. During the lesson on teen numbers,
Chen only allowed very short answers. As illustrated in the previous section, Chen only
allowed short answers to specific questions. In the second week, during her lesson on
time, Chen once again asked the children closed-ended questions. At one point, she
asked them what their favorite season was and why. Each child attempted to give an
extended answer to these questions, but each time, Chen stopped them. Chen only
allowed the children to give short answers. This was very typical of Chen’s behavior in
the first two weeks. When a child attempted to give an extended answer, Chen cut the
answer short. The pattern in these first two weeks was for Chen to speak extensively and
then to allow the children only short answers. The children responded to this rule, and
the other child-centric rules, with frustration and expressions of boredom. Furthermore,
even though Chen kept reminding them to give short answers only, the children persisted
in giving lengthy answers. This persistence eventually persuaded Chen to allow lengthier responses in the next phase.

Although she required short answers from the children, Chen spoke for long periods herself. After giving the children a lengthy talk on the seasons and what caused them, Chen announced that she would call on people with their hands raised and ask them what their favorite season was.

She called on Hannah first. Hannah said that her favorite time was Halloween. Hannah then stated that fall was her favorite season because she loved Halloween. She then began to share that last Halloween, her cousin Kaitlyn had a fever and could not join her to go trick-or-treating. Chen asked her to save this longer story for later.

Chen moved on to Nancy, who had her hand raised. Hannah protested that she wanted to finish her story now. Chen ignored this protest. Later, in reviewing the transcripts, Chen observed that she could have used Hannah’s story as an opportunity to talk about how fall is the beginning of cold season. She reasoned that this connection may have helped solidify concepts about time in her students’ minds. On this and on other occasions, Chen began to see potential in the children’s answer for the betterment of her lessons.

In the lesson I have described above, Chen’s rationale for keeping children’s contributions short was that she wanted everyone who was raising their hand to get a chance to speak. She also wanted to leave time for a kinesthetic activity she had planned for the end of the lesson. In the first two weeks, Chen did not want to sacrifice any part of her pedagogical agenda for the sake of preserving the integrity of the children’s
conversation. Chen valued her lesson plan more than she did the children’s conversation. Her instruction was teacher-centered and not developmentally appropriate.

Children had to raise their hands to speak. During the first phase of the study, Chen insisted that children only speak when she called on them. Children had to raise their hands and wait for Chen to call on them. Children who talked out of turn without being called on were quieted even if the contributions they offered were correct or potentially interesting. It is important to note that even though Chen enforced this rule, the children still constantly called out and only raised their hands each time Chen reminded them to. As a pattern, the children repeatedly broke Chen’s rules in spite of her attempts to strictly enforce them. Chen was not aware of this pattern until she saw it clearly emerging out of the transcripts.

During the first lesson on teen numbers, Chen put the number two over the numeral zero on the number ten. Logan was raising his hand, so she called on him. She asked him what number ten and two made. Logan answered, “A hundred”. Mahesh spoke out in protest. Mahesh said that actually the answer was twelve. Hannah called out as well, stating that the right answer was twelve. Chen rebuked these two children (both of whom she considered gifted) for speaking without being called on. She reminded them that they had to respect the person was speaking, and that it was Logan’s turn to speak. Chen turned back to Logan and bade him count out the Unifix cubes. Logan made a bar of 10 and a bar of two with Chen’s verbal assistance. When he counted the two bars together, he came to the correct answer of twelve.
In the interview I conducted with Chen at the end of the week, she considered that she should have said something to validate Mahesh and Hannah’s correct answers, especially because so few children gave correct answers during this lesson. Further, she felt she was not serving the needs of these two children that she saw as being gifted. Reflecting upon the transcript of the lesson, she wondered how she could have supported Mahesh and Hannah in discussing with Logan how they came to the right answer. She expressed the desire for the children to converse with each other, but she was not sure how to support such conversation without the children getting out of control. She stated that she was going to think about this problem and to experiment soon with ways to engage all the children—including the gifted ones—in more conversation.

One person spoke at a time. Along with the imperative that children only speak when called on, Chen also enforced the imperative that only one person speak at a time. The exception to this rule occurred when Chen prompted the children to answer in chorus. In a lesson on teens, when Chen had called on Logan to count out 12 using the Unifix cubes, Logan struggled to count out 1 correctly. At first, he made a bar of eight and then stated that it was ten. Beth protested, telling Chen that Logan had counted wrong. Beth said Logan had only pointed to eight even though he verbally counted to ten. Many of the children possessed enough math knowledge to recognize when a peer had given a wrong answer. It was irresistible for these children to call out to correct their classmates. In the first two weeks, Chen did not honor these corrections.

Chen reprimanded Beth for talking over Logan. She reminded her that only one person should speak at a time. Later, in our interview, Chen observed that this event
could have served as another opportunity for the children to converse with each other. She was still concerned that once she let students speak out of turn, they might all start speaking at once and cacophony would ensue. Chen had not yet developed strategies for managing conversations. However, she saw that the children persisted in speaking at once and without raising their hands. She would develop conversation strategies in the weeks to come to help her manage these behaviors. Only these pedagogical tools would come to make her feel secure enough to allow children to contribute more freely.

Personal connections were discouraged. In the math lesson that took place during week two, Chen’s students tried very frequently to verbalize personal connections to the lesson content. However, because Chen considered these shares to be off-topic, she stopped the children from sharing further on these occasions. She usually asked the children to wait to talk about those topics later. However, she never came back to these contributions. This was typical for the first two weeks of the study. Chen did not honor the children’s personal connections. Chen’s refusal to allow children to make personal connections was quintessentially teacher-directed. Without allowing children to make personal connections, Chen’s instruction could not be said to be developmentally appropriate (Copple & Bredekamp, 2010). However, Chen eventually took note that the children were especially eager to share personal stories. In fact, this was the most persuasive aspect of the children’s behavior in the first phase. Chen noticed in reviewing the transcripts that the children were actually desperate to share their personal stories in connection with the material. This desperation gave Chen pause. She realized that she had to give children a chance to do what seemed to come so naturally to them.
In the second week, when Chen was talking about morning, noon, and afternoon, Nancy spoke out of turn saying that her mother woke her up very early in the morning, and that she always wanted to sleep more. Chen ignored this comment and went on to call on Elodie, who was raising her hand. She asked Elodie if morning time happened when our side of the earth was facing the sun or when our part of the earth was facing away from the sun. This example is illustrative of Chen’s tendency to ignore personal connections in order to stick to her pedagogical agenda.

Chen continued to ask the children closed-ended questions, leading them to confirm that when their side of the earth was facing the sun, it was day time, and that when their part of the earth was facing away from the sun, it was night time. Olivia called out and said that she liked staying up late at night, but that her mother always made her go to bed early. Chen discouraged this personal connection by telling Olivia that it was not the time to tell stories. Chen stated that it was time to answer her questions. She reassured Olivia that there would be time to talk about personal stories some other time. As was typical in the first two weeks, Chen failed to come back to this topic at a later time.

I perceived that Chen stopped children from telling personal stories during the whole group lesson for the same reason that she stopped them from speaking out of turn or from speaking at the same time. She was worried these children would send the conversation in a chaotic direction. However, reviewing the transcripts in the subsequent phase (new rules for conversation), she reconsidered her position on children’s personal
connections and decided that they were exceedingly important. Only then did her instruction become developmentally appropriate.

Throughout the first two lessons, children expressed frustration when Chen stopped them from talking. Some of them acted out by making a disruptive noise, by lying down on the carpet, or by starting to play with whatever toys were within their reach. Some stated plainly that they were not done talking and that it made them mad. Observing the children in the first two weeks, it was obvious that they were not enjoying the math lessons, nor were they engaged. In interviews, Chen stated that she had always considered this state of affairs as normal:

- It upsets me when the children act up. Some years are easier than others. I guess I just think about it like, I have a difficult group this year. I think to myself, they’re not quite developmentally ready for what I’m teaching them. I always think it’s just them. That’s why we struggle. . . . And we can’t . . . there’s nothing I can do about it. But I’m reading these transcripts you gave me and I’m seeing it . . . I’m starting to see it differently. Maybe it’s something I’m doing.

Or something I’m not doing.

The picture of the children’s malaise began to emerge from the transcripts as Chen was reviewing them. Chen acknowledged that she was unprepared to engage her students in conversation. Her lack of preparation resulted in many lost opportunities for conversation during her math lessons. She came to acknowledge that her teaching practices were causing the children to feel bored and frustrated. She ceased to blame the children and she took on the responsibility herself. She began to generate rudimentary
ideas about how she could change her instruction, especially vis-a-vis conversation. Although she did not use this phrase herself, she had begun to think about ways she could engage children in developmentally appropriate conversation.

Linguistic minority students were excluded. Aarav and Forrest were the two students in the group who were in the linguistic minority. At home, Aarav spoke Hindi. Forrest spoke Mandarin at home. In the first two weeks of the study, Chen only called on Aarav once and on Forrest once. The reason she gave for not calling on them was that they did not raise their hands except in one isolated instance when Aarav did so. During these first two weeks, Chen was, for the most part, following the unspoken rule of only calling on children who had their hands raised. As described in other literature on the topic, in the first two weeks of the study, these children who needed the most practice using academic language received the least (Lagrander & Reid, 2000).

In a lesson on teen numbers, Chen asked the children who wanted to put the card that bore the number six on top of the zero on the number ten card. Aarav raised his hand. This was the only time in the first two weeks that Chen called on Aarav.

Later in that same lesson, Chen asked the children what number came after the number eight. On this rare occasion, Chen called on Forrest even though he did not have his hand raised. She asked Forrest to tell her what number came after eight. Forrest hesitated. Chen encouraged Forrest by telling him he could do it. Forrest answered that nine came after eight. Chen praised him for giving the correct answer. This was the only time during the first two weeks that Forrest spoke during the whole group lesson. This
presented a negative case in the context in which linguistic minority children were excluded. However, this negative case was an isolated one.

For the most part, Chen did not call on students in the linguistic minority in the first phase of the study. In the second and third phases of the study, Chen came to converse far more often with Aarav and Forrest although they continued to talk less than most of the other students. Overall, Chen made less progress in conversation with these students than she did with the others. This lack of progress presented a challenge to my theory that the key to productive conversation with children of this age was developmentally appropriate practice. In the case of Aarav and Forrest, cultural sensitivity may also have been key. Chen was at a loss for how to sufficiently differentiate instruction for these children even by the end of the study.

New Rules for Conversation: Weeks Three and Four

In the beginning of the study, Chen described herself as having some skill in supporting pre-math and mathematical conversations. As I observed her, however, I found that she taught in a teacher-directed style (Tzuo, 2007) that permitted little conversation among the children. Her instruction was not child-centered or developmentally appropriate. As described above, she was inspired by an in-service speaker who visited the school a month before the study began. Additionally, over the course of the first two weeks of the study, Chen reviewed the transcripts of the lessons which I gave her at the end of each week. Chen read these transcripts and made notes in them before we met for our interviews at a cafe close to her house. This review and note-taking was an intense process for Chen. Over the course of this reflection, she became
committed to changing her practice to allow the children’s voices to be heard. She came
to see herself as an unprepared pedagogue. Happily, in the second phase of the study,
Chen was able to draw upon her own resources to develop strategies, even in the absence
of explicit training.

In this phase of the study, Chen overcame many of the obstacles that stood in the
way of conversation in the first phase. She still wanted to get through her pedagogical
agenda on any given day. She feared the children’s off-topic contributions in case these
might derail the conversation. Finally, she feared teaching differently than her peers.
However, she overcame these obstacles by facing her fears, by accepting more student
contributions, and by using certain conversation strategies. Chen specifically spoke of
the bravery that would be required of her to change her practice:

It’s not an easy thing for me to change what I’m doing. I’ve been doing the same
thing for a long time. I have reasons. I don’t know what people are going to say
or if the kids are going to learn . . . if I change, I mean. Like I’ve said before, I
have to toughen up and be brave. I think these kids deserve it. I honestly think I
can do better. And I mean I won’t be sure unless I try. If I try something and it
doesn’t work, I’ll just try something else!

Chen’s ability to overcome these obstacles will become clear as I describe the
new rules for conversation that she put into place during this phase. The obstacle the
children posed by offering off-topic contributions ceased to be an obstacle in this phase
because Chen made use of these contributions and showed how they were, in fact, on-
topic.
Another obstacle the children posed also ceased to be a problem in this phase. In the first phase of the study, the children were sometimes unable to understand the lesson content. Now that Chen involved them in conversation, the children were far more likely to understand the lesson content. Thus, in the second phase, Chen helped overcome obstacles that both she and the children were posing. This eradication of obstacles could be the reason Chen only needed to resort to classroom management 14 times in the second phase compared with the 38 times in the first phase.

Chen developed new teaching strategies in her planning time as well as on her toes as she worked with the children. Along with the memory of what she had learned in her master’s degree program and the memory of the recent in-service, the transcripts proved to be a powerful tool that helped Chen reflect upon and later change her practice to incorporate more conversations in her lessons. Although Chen still felt basically unprepared to engage children in pre-math and mathematical conversations, she planned to “bravely” conduct pedagogical experiments. These experiments served to prepare her to engage children in productive conversation. Thus, Chen prepared herself for pre-math and mathematical conversations. All of the new rules described in this section were experiments on Chen’s part and required “bravery”, as she phrased it.

As Chen reviewed the transcripts during the first two weeks, several things caught her attention. She noticed that she consistently stopped children from sharing personal stories. She reflected on this pedagogical decision on her part and realized that in stopping the children from making personal connections, she could be stopping the children from internalizing the contents of the lesson. She recalled from her master’s
degree program that children accessed instruction by making personal connections to it. Although Chen did not verbalize this fact, it was developmentally appropriate for children to make personal connections and enjoy freedom to make connections to their prior knowledge (Copple & Bredekamp, 2010). Children of this age should be the ones to decide how to connect to the content being taught. To forbid them to make these connections was to deprive them of access to the lesson. In her own way, Chen acknowledged these facts. She decided that in the future, she would try to see the connections between the children’s personal stories and the contents of the lessons. To engage in this pedagogical experiment, Chen had to overcome her fear that the children would derail the conversation.

Chen noticed that when the children spoke, it was usually to make a personal connection. Thus, in stopping children from making personal connection, she was preventing any conversations from being extended. She realized she was not “striving for five” as the speaker had encouraged them to do. I observed that in stopping the children from speaking, she was preventing them from co-constructioning sociomathematical norms. The children remained peripheral participants in the classroom micro-community, far from participating like Chen in a central manner. By stopping the children from contributing in the way they saw fit, she was also stopping the children from taking ownership over their learning.

Chen noticed in the transcripts that she had the habit of quieting children even when they called out correct answers. Chen noticed that she was not including all the children in the conversation. Her unspoken rule dictated that she only called on students
who were raising their hands. Thus, those children who did not raise their hands did not get to participate in the conversation. The children who rarely raised their hands included the two students who were in the linguistic minority. Chen did not verbalize this idea, but by only calling on children who were raising their hands, she was not offering equitable opportunities for children to move into more central participation in the classroom micro-community. Although I did not share any concepts from my theoretical and conceptual framework with Chen, she was aware that her instruction was not equitable.

Chen decided to undertake a pedagogical experiment and let the children speak without raising their hands as long as they did not all talk at the same time. She also resolved to call on children even when their hands were not raised. She reasoned that if these pedagogical experiments did not work, she could always try something else:

I’m going to be brave and do some things I’ve never done before. These are big changes for me! I’m always drilling them to raise their hands before they can speak. I’m a big, like, what would you say? I’m a big advocate for self-control. I still want the children to practice self-control, but I have to do something different during these math lessons. So I’m going to let them talk without raising their hands. I’m going to make a bunch of changes and see what happens.

Unlike in the first two weeks when Chen was not aware that she enforced rules to govern conversation, now Chen was consciously implementing new rules for conversation. Chen was edified by what she had seen in the transcripts during the first two weeks. She had seen that she was not engaging the children sufficiently in
conversation. She also saw that the children were acting out and breaking the unspoken rules. She realized that new rules were in order to make her instruction more appropriate for her preschool-age children. Some of these changes Chen talked about explicitly with me as well as with her students. By making her expectations for children’s conversation more explicit, she gave children a better chance at being successful in engaging in conversation (Walshaw & Anthony, 2008).

Other rules she enforced without stating explicitly that she was doing so. First, I describe her explicit rules for conversation. Then I describe her implicit rules. As stated above, Chen had long desired to teach in a different way. However, she did not know how or where to start. She was afraid that the administrators or parents would dislike the changes she might make. She stated, however, that she felt protected by her participation in a doctoral study. For the first time, she made changes that she perceived were daring. These changes required bravery on her part.

Explicit Rules for Conversation

There were a number of new rules for conversation that Chen talked about explicitly with me and with her students. These rules stood in stark contrast to the unspoken rules of the initial verbal interaction phase. Chen’s stated goal for these new rules was to include more of the children in the premath and math conversations. Chen began to engage children in extended conversation wherever possible. Chen now encouraged the children to talk to each other. She said that it was okay for children to disagree with each other or with the teacher. Children could request to speak. Children could speak without raising their hands. Finally, the teacher explored student reasoning.
Chen enjoyed some success with these new rules. She expressed that she felt successful because she was including more of the children in the conversation. However, at the end of this phase, she still felt she had work to do to engage the children more completely in conversation. She still felt like the unprepared pedagogue, and yet she knew she was moving in the right direction. Most of all, she was proud that she made progress by drawing on her own pedagogical imagination.

Teacher engaged children in extended conversation. Chen started to change her practice by validating students’ contributions instead of stopping them. Her new goal was to engage the children in extended conversation. Even when children offered an answer that was incorrect, Chen did not shut them down. Rather, she encouraged other children to respond to these contributions. This was a dramatic change from her practice in the first phase of the study, and it required a great deal of bravery on her part. She was still afraid that conversations would get out of hand. Only experiences could teach her otherwise. In the third week of the study, Chen taught a second lesson on the topic of time. She started the lesson by telling the children explicitly that she wanted them to engage in conversations. She added that these conversations could be long—as long as the children wanted.

Next, she initiated a premathematical conversation about a toy clock she held in her lap. Early in the lesson, she pointed to the hands and asked the children what they call these things. Emily pointed out that her mother had those on her watch too. Chen did not reprimand her for speaking out of turn or without raising her hand. Rather, she extended the conversation by asking Emily if she knew what those things on her
mommy’s watch were called. Chen let a number of other children answer, and she commented on each child’s answer without telling them they were wrong. Olivia answered that they were called turners. Chen validated Olivia’s answer by saying that they did indeed turn. Emily guessed the hands were called wires. Only after exchanges with several students did Chen reveal that these parts of the clock were called hands. Further, she said, one was a minute hand and the other was the hour hand. Reflecting later on the transcripts, Chen was pleased to see that here and at other times, she had elicited answers from many children before revealing the correct answer. She was pleased that she had engaged more children in conversation. She noted happily that the conversation did not get out of hand as she had feared it would.

After giving this explanation about the long and short hands on the clock, Chen talked explicitly about the conversation they were having, telling the children that she was pleased that they were able to go back and forth. I would argue that this explicit explanation of conversation was developmentally appropriate. Chen did not assume that the children were sophisticated or knowledgeable enough to know what comprised a productive conversation. Here and throughout the weeks that followed, Chen provided explicit description of productive conversation. These explanations ensured that the children could participate regardless of their level of development. Chen pointed out that going back and forth is how you have a good conversation. She also added that she liked that many people gave answers and that she wanted everyone to join in the conversation. By making her expectations for the children’s conversation explicit, she made it more
likely that the children, and especially students in the linguistic minority, would be more successful in practicing academic language.

In the fourth week, Chen read a book by Eric Carle called *The Grouchy Ladybug*. After reading the book and talking about what the grouchy ladybug did during each hour of the day, Chen engaged the children in a conversation about what they did during each hour of the day. Chen put the book away and took out a toy clock. Chen reminded the children that they were practicing holding conversations. She told the children that she wanted to hear them talk about what they did at each hour of the day. Chen stated, “We can talk all about it...and we can talk for a long time if you have a good idea worth talking about.”

After this explicit reminder about what it meant to hold extended conversation, Chen engaged in extended exchanges with several students as they defended their reasoning for guessing what they did for each hour. Chen expressed to me that she felt successful in extending conversation with these children and also in deepening the exploration of premath and mathematical ideas.

Chen also tried engaging Forrest, one of the students in the linguistic minority, in an extended conversation. Forrest spoke up guessing that he went home at four o’clock after snack time. It was the first time that Chen could remember that Forrest made a comment unsolicited. It was likely that Forrest made this comment as a response to Chen’s verbalization of her expectations of the children’s conversation (Lagrande & Reid, 2000). Chen asked him follow-up questions to extend the conversation with him. However, she was unsuccessful. Forrest shrugged his shoulders in response to her further
questioning. It was notable that not all Chen’s attempts at extending conversation were successful. During the second phase of the study, linguistic minority students were still unlikely to contribute to conversation in an extended manner. In addition to making attempts to extend conversation, the teacher encouraged her students to engage in conversation with one another.

Children were encouraged to talk to each other. The children were accustomed to addressing Chen exclusively during all verbal exchanges in the first two weeks. However, as Chen reflected on the transcripts and on the true mathematical needs of her students, she decided to experiment by encouraging them to communicate with each other more and not just with her. She reasoned that if they were to enter STEM (science, technology, engineering, and mathematics) jobs, it would be necessity for them to converse with their peers. At the beginning of the third week’s lesson, Chen told the children that she wanted them to engage in conversations. She specifically requested that the children converse with each other. Chen promised to help the children with this new challenge. Once again, making her expectations explicit, she made it more likely that the children would be successful in practicing academic language (Walshaw & Anthony, 2008).

The literature on developmentally appropriate practice (Copple & Bredekamp, 2010) did not explicitly describe the importance of children’s mathematical conversation with each other. However, it did emphasize the importance of children’s practice socializing together as well as the importance of early academics. Once Chen engaged the children in conversation with each other, the children were noticeably more happy and engaged. They also remembered more of the material. These results suggested that
children’s conversation with peers during math lessons was indeed developmentally appropriate.

During week four, Chen asked the children what they thought they would be doing at five in the morning. Logan answered that he would be brushing his teeth. Erynn answered that she would be eating her breakfast. Mahesh answered that he would still be sleeping. Chen asked Mahesh to explain his thinking to Logan and Erynn. As was typical of Chen in the second and third phase of the study, she specifically asked that Mahesh address his peers. Mahesh turned to Logan and Erynn and said that the alarm on his table said seven o’clock when he woke up and he knew that five came before seven. Thus, he concluded, he must still be sleeping at five o’clock. In this instance, the teacher was not the only mentor to her students. Mahesh acted as a more knowledgeable member of the microcommunity and mentored his peer (Lave & Wenger, 1999).

Chen became skillful in scaffolding the children’s mentorship of each other. She often gave the students she considered gifted this role so as to offer them a much-needed challenge. Mahesh possessed the expertise needed to mentor his peer, Logan. However, that did not mean that Logan would be happy to receive this mentoring. In fact, Logan resisted being taught by his peer. To make sure Logan felt heard, Chen gave Logan a chance to explain his thinking to Mahesh in return. By encouraging children to converse together, Chen ensured that the children would be more receptive to the mentorship of one of the gifted students. In this particular example, after asking a few other children about their thoughts, Chen taught the students that they were all probably asleep at five o’clock. With Chen’s assistance, the children were successful in addressing each other.
However, in this phase of the study, they were not ready yet to address each other without prompting. Also, students like Logan had not yet accepted direct talk with a peer as a norm. Another strategy Chen used was to allow the children to disagree with one another. Allowing children to disagree eventually helped them to move towards establishing a norm.

It was okay for the children to disagree. Upon reflecting upon what would help prepare the children most for complex mathematical work later on down the line, Chen decided that she should teach the children that it was okay to disagree and to have to explain their reasoning to each other. In the third week’s lesson, when Chen introduced the concept of conversation to the children, she explicitly told them that it was okay to disagree with a friend about ideas during the math lesson. She made talking mouths with her two hands. She made one hand say to the other, “I think one plus one is two.” Then, pretending the other hand spoke, she said, “I disagree. I say one plus one is three.”

The children laughed, demonstrating that they were listening. Chen asked the children how they could settle this disagreement. Hannah answered that they could use the Unifix cubes or another kind of math counter to find out. Olivia pointed out that you could just use your fingers. During this lesson, there were no disagreements among the children. However, in the fourth week’s lesson, there were several, and Chen helped the children to navigate all three disagreements by verbalizing their reasoning to each other. I have already described the first disagreement above. To reiterate, when Chen asked what the children would be doing at five o’clock in the morning, Logan, Emily, and Mahesh all gave different answers. Logan became upset about this disagreement. Chen
prompted Mahesh to explain to Logan and Emily why he thought he would still be sleeping at five in the morning.

Mahesh gave his answer, but Logan was still upset that Mahesh disagreed with him. To appeal to Logan, Chen prompted him to give his own explanation for his answer, and to address Mahesh directly. Logan told Mahesh that his mother told him it was five o’clock when he was brushing his teeth. Chen validated this answer although it sounded far-fetched. Chen conceded that she was not at Logan’s house in the morning, so she had only Logan’s account to go by. After Chen allowed him to give his explanation, Logan appeared less upset, and Chen continued with the conversation. Chen was pleased that in this example and in others, she had offered the children an opportunity to be involved in the conversation, and specifically to be able to talk to each other. In scaffolding the children in talking through disagreements, she was pushing them to the edge of what would be considered developmentally appropriate. She was pushing the children at the top of their ZPD (Vygotsky, 1934/1978) for conversation.

It was clear that the children were being scaffolded at the top of their ZPD because they required a great deal of assistance in order to engage in this kind of conversation. It was a challenge for them, but with help from Chen, they overcame this challenge. Chen noted in the following interview that she would like for the children to talk to each other without her prompting. As the children practiced talking through disagreements over the course of weeks three and four, they became visibly more comfortable with this conversation practice. This ease helped Chen implement another strategy: allowing the children to request to speak.
Children could request to speak. In many instances in the first phase of the study, children explicitly requested to speak. In each of these instances, Chen told these children “no” or to wait until later. But in the latter cases, she never came back to that child to let him or her contribute. In the new rules for conversation phase, however, when a child requested politely to speak, Chen allowed him or her to contribute as soon as he or she wanted to. She overcame her fear of the children’s free contributions.

In the third week, Chen was conversing with the children about clocks and where clocks could be found. Emily asked Chen if she could say something. Emily did not raise her hand before speaking. Chen said, “I’m going to let Emily talk because she asked twice to talk. She asked politely. Thank you for saying that instead of interrupting. Yes, Emily?”

Through explicitly condoning a child’s request to speak here and in other instances, Chen made her expectations explicit (Walshaw & Anthony, 2008). Once Chen allowed her to share, Emily stated that she saw the time on her mother’s phone. This led to other children sharing that they saw the time on their parents’ phones. Later in the lesson, when Chen wrote all the time-related vocabulary words the children could generate on a big piece of paper, many of the children remembered and offered the word, “phone”. Later in the same lesson, Mahesh asked if he could say something. Once again, he asked without raising his hand. Chen allowed him to speak. Mahesh shared that his brother had an alarm clock on his dresser. The word “alarm” was another word the children remembered when Chen wrote time vocabulary words on the big sheet of paper. By allowing children to speak when inspiration struck them, Chen was giving her
preschool-age children freedom in how they were to make connections to the material. This freedom granted the children more access to instruction. Their conversations became more developmentally appropriate.

Before that week’s interview, Chen compared the transcripts of the third lesson with those of the first two lessons. She observed that the children seemed to remember very little of the lesson content during the first two lessons. However, in the third lesson, when she allowed them to engage in more conversation, they were able to remember the lesson content. In fact, it seemed to her that the more a child conversed during the lesson about time, the more words they were able to remember at the end during that informal assessment on time vocabulary words. Additionally, allowing children to speak when they requested led to engagement of more children in the conversation.

Children could speak without raising their hands. During the initial verbal interactions phase of the study, Chen insisted that children raise their hands and wait to be called on. However, upon reflecting on these early transcripts, she was dismayed that this practice clearly stopped conversation from flowing. Chen was afraid that by allowing the children to speak without raising their hands, they might all interrupt each other and start to fight. However, her goal of encouraging conversation was so important to her that she decided to relax this rule as a pedagogical experiment. During the new rules for conversation phase of the study, Chen allowed the children to speak without raising their hands. Chen was pleasantly surprised that the conversation did not get out of control. The children only interrupted each other on a few occasions, and Chen was able to manage the conversation in these instances.
On one occasion, Chen told the children that her husband had the same alarm clock for 20 years. Olivia asked Chen how old her husband was. After Olivia asked this question, the children began chattering about how old their parents were. Chen reminded them gently that they did have to listen to the person who was talking even if they were allowed to speak without raising hands. The children listened to Chen and turned their attention back to Olivia. Chen successfully managed this and other occasions when children all spoke at the same time.

In fact, Chen came to recognize that those occasions when the children all spoke at once were great conversation opportunities. When they began to chatter together, they were obviously excited about the ideas they were telling each other about. Chen learned to call on a few children to say more about what they had been chattering about. Usually, calling on a few children to share was enough for Chen to help the children feel they had been heard.

At the beginning of week three’s lesson, when Chen introduced the idea of conversation to the children, she let them know that they could speak without raising their hands. She made the point that they should still make an effort not to interrupt each other. She told them that she wanted them to listen to each other, and that was all she asked. During that lesson, Chen was true to her word and allowed the children to speak without raising their hands. For example, when Chen pointed to the hands of the clock and asked the children what they were called, several children called out answers. They answered, “turners”, “wires” “arrows”, and “arrow movers” among other answers. Their answers were incorrect, but Chen was happy that they enjoyed practice engaging in
conversation. Chen did not correct them right away. Instead, she waited until the children gave all the answers they could generate. Then she told them their answers all made sense but that the objects were actually called “hands”.

Once the hand-raising rule had been relaxed, Chen used the opportunity to call on the students who were in the linguistic minority. Thus, Chen began to disrupt the cycle of inequality in which the children who needed the most practice with academic language enjoyed the least. At the beginning of the third week’s lesson, Chen and the children were talking about clocks and what shapes they came in. Chen summarized what the children had been saying by stating that some watches were circle-shaped while others were square- or rectangular-shaped. Then she turned to Aarav and asked him where in his house he saw the clock.

This attempt at including Aarav in the conversation was not successful. He did not answer. Close to the end of the conversation, Chen called on Aarav again. Once again, she called on him even though his hand was not raised. Chen had been talking to the children about how they knew what time it is. She called on Aarav and asked him what told him the time. Aarav did not answer right away, but Chen persisted. She asked him again, “How do you know what time it is?” Finally, Aarav answered that he knew by looking at the clock. Then Chen engaged him in a somewhat extended conversation about times of day when Aarav uses the clock. In this instance, the relaxing of the hand-raising rule resulted in more involvement in conversation by a student in the linguistic minority.
Several times during the second phase of the study, the students in the linguistic minority spoke when called on. However, Chen was not satisfied. She wanted the students in the linguistic minority to engage in more conversation, especially without prompting. This did not occur, however, until the final phase of the study. Even by the end of the study, the students in the linguistic minority spoke up less often than the students in the linguistic majority. This lack of involvement of linguistic minority children in conversation was a negative case that challenged my theory that developmentally appropriate instruction was key to engaging children in productive premath and mathematical conversation. It seemed that these children required a different intervention. Chen was not able to discover the ideal interventions for these children even through her frequent pedagogical experimentation. However, as I will discuss next, Chen’s new strategy of exploring student reasoning revealed how children were thinking through the problems that were presented to them.

Teacher explored student reasoning. During the first phase of the study, Chen was rigid in the answers she accepted. She required that students answer on topic. If their contribution seemed off topic, Chen stopped them, sometimes mid-sentence. However, after reflecting on the transcripts, she decided to let her students to talk more even if it meant their answers would be off topic. Chen noticed in her review of the transcripts that some of the seeming off-topic responses could actually have been connected back to the topic at hand with some discernment on her part. Thus, Chen resolved to accept all answers and to explore the students’ reasoning that connected their contribution to the premath or math topic at hand.
At times, Chen was creative in connecting the children’s contributions to the topic at hand. She had to creatively explore students’ reasoning in order to do so. As a result of this practice, the conversation flowed more easily and the children took greater ownership over their contributions.

During the third week, Chen asked the children what helped them tell the time on the clock. She was hoping someone would say “the numbers”. Olivia asked if she could speak and Chen gave her permission to do so. Olivia said simply that she woke up. Chen asked her if she knew what time it was when she woke up. Olivia said simply that her mother woke her up. Chen asked again if she knew what time it was. Olivia said that it was still night time when she woke up. Chen asked Olivia how she could be sure it was night time. Olivia said that it was dark out and that that was how she knew. She stated that they lived so far away that they had to get up when it was still dark out. Chen turned to the other children and told them that Olivia brought up a good point. Chen said that when it was very early in the morning, like five or six, sometimes it was still dark outside. She turned to Olivia and told her she was really waking up in the morning. It was just very early in the morning while it was still dark out.

During a subsequent interview, Chen reflected on this and other instances in which she would have previously stopped the children from speaking. She expressed satisfaction that now she was able to explore the reasoning behind children’s contributions. In the above example, Olivia’s contribution may have seemed irrelevant at first. However, upon exploring her contribution, Chen found that Olivia had an interesting piece of information to share with the rest of the class. This piece of
information was relevant to the topic of time. On many similar occasions, Chen found that the children’s seemingly irrelevant contributions reflected interesting reasoning on their part. Chen was not always successful in discerning a child’s reasoning; sometimes a child’s contribution was truly off-topic. However, Chen continued to strive to make these meaningful connections. In the next section, I will discuss the implicit rules of conversation Chen implemented.

Implicit Rules for Conversation

In addition to the explicit rules described above, I observed Chen to follow certain unspoken rules. As a rule, Chen asked questions to encourage conversation. Another rule was that, while Chen handed over more responsibility for conversation to the children, she remained in charge of the conversation. Chen now encouraged personal stories and she connected these personal stories to the topic at hand. The teacher differentiated conversation strategies for different students, such as for her two gifted students and her two students in the linguistic minority. When children had trouble answering a question, Chen offered them hints to help them participate more fully in the conversation. Finally, Chen successfully included all students in premath and mathematical conversations to some extent. The students in the linguistic minority still remained on the periphery of conversations during this phase, although Chen was successful in bringing them into more conversation than in the first phase of the study.

Teacher asked questions to encourage conversation. In the first phase of the study, Chen asked questions looking for a precise answer, such as in the lesson on the teen numbers when she asked the children over and over what number came next and how to
count it out using Unifix cubes. In the new rules for conversation phase, however, Chen asked questions to encourage children to answer in any way they wanted to. In the third week’s lesson, after giving an introduction to the concept of conversation, Chen showed them her toy clock and asked them simply what it was. The children did not give the answer right away. Rather, several children said that they had one at home. Others pointed to the real clock hanging on the wall and said they were the same thing. Finally, Beth said that it was a clock and it could go around and around and around. Chen told her she was correct and that it was a clock. She noted also that everyone was correct in saying they had one at home and that they had one on the classroom wall. Then Chen pointed to the hands and asked what those were called. Once again, the children were free to answer in an open-ended way. The children had more freedom to answer in the way that made sense to them. This new practice was developmentally appropriate.

In the fourth week of the study, after reading *The Grouchy Ladybug*, Chen asked the children what they did at different times of day. She allowed them to answer in whatever way they wanted. She hoped they would converse with each other as they talked about different things they did at each hour of the day. As described above, Chen, Logan, Emily, and Mahesh engaged in an extended conversation after Chen asked them what they did at five in the morning. A few minutes later, Chen asked the children what they did at seven in the morning. Several children stated that they woke up at seven in the morning. The children conversed with each other, excited that they had this in common. These were just two of many instances from this phase in which Chen’s
questioning led to conversation between Chen and the children or between the children themselves.

The teacher remained in charge. Although Chen’s goal in the new rules for conversation phase was for children to speak freely, she still stepped in to structure the conversation when needed. While it was developmentally appropriate for Chen to give her students freedom to speak, she still had an important role in stepping in to offer information and to scaffold further conversation. For example, when Chen pointed to the hands on the clock and asked what these were called, her goal was for as many children to contribute answers as possible. While the children were answering, Beth interjected that she had something to say. Chen told her that at that moment they were answering the teacher’s questions.

Later in the lesson, Chen asked the children how the clock helped people to tell time. The children were in the midst of answering in an open-ended fashion when Olivia interjected that she had a question. Once again, Chen pointed out to her that they were answering her question at that time. While this scene is reminiscent of Chen’s practices in the first phase of the study, Chen only stopped children from contributing rarely, and she did so for a deliberate pedagogical reason. Chen wanted all of the children to have a chance to answer her question, and they did. It is important to note that Chen was still giving children more freedom in choosing how they were to answer her question. In the same vein, Chen encouraged personal stories from the children during the second phase of the study.
Teacher encouraged personal stories. In the first phase of the study, Chen cut short any personal stories the children tried to tell. Chen admitted that she was afraid if she let the children tell all their personal stories, this would derail the conversation so that they would be unable to cover all of the topics in her pedagogical agenda. However, when she reviewed the transcript, she noticed that when children spoke up, it was almost always to share a personal connection. Chen reasoned that this must be the only way the children knew how to access information. Even as an unprepared pedagogue, Chen was able to discern this fact on her own. In this and many other instances, Chen drew upon her own resources and used her common sense to devise new ways of enhancing classroom conversation. She considered that by preventing students from making personal connections, she was preventing them from internalizing the content. Chen still had misgivings about allowing children freedom to make personal connections. However, she was committed to changing her practice. As a result of this commitment, she decided to overcome her fear and allow the children to share their personal stories as a pedagogical experiment. She spoke again in an interview about bravery.

I still have mixed feelings about these experiments I’m doing with the children’s conversation. I feel proud about what I’m doing. But part of me feels like I’m wrong to go against the grain. I just tell myself to be brave. It doesn’t make sense to keep teaching the same way if you’re not getting the result you want. I know for sure the kids should be making personal connections. That’s a realization I’ve had.
Chen was surprised to find that the conversation was rarely derailed when children shared personal stories. Chen found that she could usually creatively connect these stories back to the topic at hand. She realized that it was up to her to develop more skill in orchestrating meaningful conversation so that children could share in authentic ways without going off in too many directions.

In a lesson during week four, Chen asked a question that led the children to make personal connections to the material. After she finished reading a book about time, she asked the children to tell her what they did at different times of day. She held a toy clock, and going around, pointing the hands to different hours, she asked the children what they did during each time. When Chen got to four o’clock in the afternoon, she listened to the children’s guesses before telling them that at four o’clock they normally eat afternoon snack. Olivia told Chen that she wished Chen would give them more snack because she was always hungry in the afternoon. Chen expanded upon Olivia’s story about this personal experience by saying Olivia woke up earlier in the morning than her friends because she lived far away. Chen stated that that was why she did give Olivia extra snack, since she ate breakfast earlier than her friends and ate dinner later. Chen connected this personal share to the topic of time by saying, “That’s a good connection to the topic of time. Olivia wakes up earlier than the rest of you and she stays later than you. Does that mean she’s at school for a shorter time or a longer time?”

“A longer time!” the children answered.

In this and other instances, Chen no longer stopped children from sharing personal stories. Previously, she may have considered Olivia’s contribution to be off-
topic. Now, she encouraged Olivia and other children to share personal stories, often pointing out what made them relevant to the topic at hand. She did so for purely experimental reasons, because she knew she had to do something differently. Once she made this complete turnaround in her instruction, she was surprised to find that it led to mathematically productive conversation. This change to her instruction, perhaps even more than any of her other changes, ensured that her practice was developmentally appropriate. Children must be free to make personal connections in order to internalize new material.

Teacher differentiated conversation. During weeks three and four, Chen also guided the ongoing conversation in the classroom to include children from a linguistic minority background as well as gifted children. I explore such differentiated conversation in this section. In phase one of the study, Chen noticed in the transcript that not all the children were benefiting equally from the verbal exchanges. Specifically, she found she was unwittingly excluding students in the linguistic minority (Aarav and Forrest). Also, she saw that she was not adequately challenging the two children in her class that she considered gifted (Hannah and Mahesh). In phase two, Chen made plans to engage Aarav and Forrest even when they did not raise their hands. She also planned to let Hannah and Mahesh teach their classmates the things they knew. Chen believed this would offer them further stimulation. In this way, Hannah and Mahesh could act as more knowledgeable peers and mentor their fellow students (Lave & Wenger, 1999).

Above, I have given examples of ways Chen tried to include Aarav by calling on him even when he did not raise his hand. She did the same for Forrest during the second
phase of the study. Chen did not have to call on most of the other children in order to elicit verbal contributions from them, but she differentiated her conversation style for Aarav and Mahesh by calling on them even when they did not volunteer to speak. In the conversation they had after reading the book in the fourth week’s lesson, Chen asked the children by show of hands who thought they woke up at seven o’clock. Many of the children raised their hands. Chen noticed that Forrest was not raising his hand. She called on him and asked him if he also woke up at seven. Forrest nodded his head and said yes. Later in the conversation, Chen asked the children what they did at twelve o’clock. Forrest spoke without being called on to say that he thought they ate lunch at twelve. Chen was very pleased that Forrest volunteered an answer without being called on. In this and other instances, Aarav and Forrest began to occasionally volunteer contributions. It appeared that they were emboldened to speak more often, perhaps because of the practice they had gained when she called on them. Although they never spoke as much as their linguistic majority peers did, even in small group and pair work, these students did make some progress in joining in conversation.

In this same conversation, when Chen asked the children what they did at seven o’clock, she listened to their guesses. Then she asked Mahesh to teach the class about why he thought he woke up at seven. Mahesh answered that he had an alarm in his room. When the alarm went off, the alarm said seven-zero-zero. Chen called the children’s attention to the fact that Mahesh’s clock was probably different than the toy clock she was holding in her hands. She asked Mahesh if his alarm clock showed all of the numbers like her clock. Mahesh said no, it only showed the numbers for what time it
actually was. Mahesh added that his mom and dad’s phones were the same in this way. Chen used Mahesh’s contribution to tell the children that those kinds of clocks were called digital clocks.

Later, when Chen was collecting time vocabulary on the big sheet of paper, Mahesh remembered this piece of information and shared, “digital clock”. Chen stated in interview that she felt satisfied that she had allowed Mahesh to showcase his knowledge. She also pushed him a bit forward by teaching him a new word. Chen gave Mahesh and Hannah many similar opportunities to show off their knowledge as well as to learn something new. She found it was easier to differentiate instruction for the gifted children than it was for the students in the linguistic minority. She found it was easier because Mahesh and Hannah were always eager to speak. However, through continued pedagogical experimentation, Chen succeeded in including all of the students in conversation to some extent.

In developing the theory to describe the data from this study, I had to acknowledge certain negative cases. These negative cases involved the students in the linguistic minority. It was clear, in reviewing the data, that instruction that was developmentally appropriate for most of the children was not sufficient to aid all of the children. Perhaps Chen’s instruction was not developmentally appropriate enough for them. Perhaps Chen’s practice should have been more culturally sensitive. Perhaps these students in the linguistic minority simply needed more one-on-one time with the teacher in order for them to understand the concepts being taught. Chen did not engage in any one-on-one time with these students. Perhaps this mode of instruction would have
enabled these children to better access whole group conversations. More research is needed to determine what interventions are best to support students who are in the linguistic minority during preschool math lessons. Chen did not differentiate instruction enough to include these children’s voices in classroom conversation as much as she did for the other children. However, as I discuss in the next section, Chen helped the children in her classroom answer the questions that were posed to them.

Teacher gave students hints to help them answer questions. In the first phase of the study, Chen asked children questions and expected them to answer without any extra support. In phase two of the study, Chen offered more scaffolding for conversations such as when she helped elucidate student reasoning or when she helped students talk through disagreements. Another specific way she scaffolded conversation was to give children hints so that they could answer questions. In the first phase, Chen gave children the answers herself when they were at a loss. In the second phase, Chen wanted to encourage the students to speak as much as possible. Thus, when they were having trouble giving an answer, Chen offered them hints so that they could be the ones to verbalize answers. Chen became more willing to scaffold the children’s learning rather than skipping ahead to the right answer. Chen was more willing now to meet children where they were in their stage of development instead of expecting them to meet her at her standard for where they should be. Once again, Chen’s instruction was now more developmentally appropriate.

After reading The Grouchy Ladybug, Chen challenged the children to share what they did at each hour of the day. The children were not able to answer these questions
without support. Chen offered hints throughout this conversation. With her assistance, the children were able to take on some responsibility for the conversation and offer meaningful answers. In one instance, Chen asked the children what event occurred during the school day at 9:30 in the morning. The children guessed wildly. However, Chen did not tell the children directly that they were wrong. She first allowed them to answer in any way that they wanted. Then she gave them hints. She reminded the children that at eight in the morning, they arrived at school and played outside. After that, they came inside for free choice time. Chen adjusted the toy clock to make it say nine o’clock and she reminded the children that at nine, they had circle time. Then, after circle time, Chen said, they all sat down at the tables. After this narration of the morning, almost all of the children were able to answer in chorus that they had snack at nine-thirty.

Towards the end of week four, the teacher had laid the groundwork for the establishment of sociomathematical norms in the classroom by exercising behaviors that relaxed preexisting rules, inviting student contributions, and making a deliberate effort to explore student reasoning. The teacher's reflection towards the closing of phase two showed that she was ready to move towards a more democratic classroom culture in which the sociomathematical norms would be co-constructed by the teacher and the children. Her instruction became less teacher-directed and more child-centered (Tzuo, 2007).

The children appeared far happier and more engaged during the third and fourth weeks than they did in the lessons of the previous two weeks. They sat up straight and looked at Chen or their fellow students when they were speaking. The conversation did not “go off the rails” as Chen worried it would if she gave the children freedom to speak.
Unlike in the first phase of the study, the children remembered the material when Chen assessed them later. The children appeared to take more ownership over their learning in this lesson than they had in previous lessons. The student engagement that ensued from the adoption of Chen's strategies during weeks three and four paved the way for the development of the sociomathematical norms during weeks five through twelve of the study.

*Figure 2* is an illustration of how Chen conducted pedagogical experiments as a result of her self-described bravery and desire to change instruction. Chen made changes to her instruction both through reflection and through planning on her toes during instruction. Through these activities, Chen’s status changed from unprepared pedagogue to one capable of orchestrating developmentally appropriate instruction.

*Figure 2*. Process leading to developmentally appropriate conversation with students.

The teacher started out as an unprepared pedagogue. Through personal bravery, she conducted pedagogical experiments. These experiments led to reflection and planning on her toes. In these ways, she prepared herself. These activities ultimately led to developmentally appropriate conversation with her students.
Sociomathematical Norms: Weeks Five through Twelve

The transition from phase one to phase two was strenuous for Chen, but easy for the children. The transition had required much reflection, brainstorming, and bravery on the teacher’s part. The children, for their part, were ready and eager to make conversation central in the math lessons. The ease of this transition suggested that indeed this new style of conversation was developmentally appropriate for children of this age. Further, the new rules that Chen generated in phase two made the transition a smooth one. Chen was excited about the changes, but still worried. In fact, she never fully stopped worrying throughout the study. In our fourth interview, Chen shared with me:

I’m happy that I’m finally teaching them this way. I mean I can see that it’s good. Because it makes them happy. Not just that they’re happy. They’re participating! I don’t know why I keep worrying. Well, I can say this. I don’t know if it’s just in my head, but I worry about what other people are going to say. Because maybe they’re not teaching this way. Well, I know they’re not. I’m afraid of what they will say if they were to drop in. Even though they don’t even come in here!

(Laughs.)

Chen was reassured when I told her the administration had given me specific permission to observe mathematical conversations. This approval suggested to Chen that the administration condoned the children’s engagement in conversation during math lessons. Chen pondered the real motivations of her administrators. On the one hand, they had chosen a curriculum that was not very reform-inspired. They encouraged the teachers to stick to the curriculum. On the other hand, they hired the speaker for the in-
service who was very reform-minded. They also condoned my study, which took for
granted a reform-oriented mathematical teaching practice.

Chen was not sure where the administration really stood. She only knew that the director
approved of some teachers practice and disapproved of others:

   Paulette considers some teachers around here to be good teachers. Others she
disapproves of. Once she disapproves of you, well, it’s hard to get off that list. I
don’t want to be put on that bad list! Because honestly, once she . . . once she,
like, labels you, it’s hard to shake free of that. She’s kind of . . . capricious. Her
reasons for liking you are not always clear. But some of these teachers...if she has
labeled them problem teachers . . . she’ll always be checking in on you and it’s
very stressful. I’m on her good list now, so she rarely comes to observe me. I
want to stay on the good list!

   Chen referred to her worries on many occasions during our interviews. However,
she made it equally plain that she planned to forge ahead in her pedagogical
experimentations. She was afraid of the administrator’s censure, but her desire to be a
good teacher was stronger than this fear. Chen often used the word “bravery” to describe
her impetus to change in spite of her fears. She was committed to finding ways to engage
her students more fully in premath and mathematical conversations. The result of this
commitment was that the children enjoyed greater opportunities to co-construct
sociomathematical norms as a discourse community in the third and longest phase of the
study. For her part, Chen broke out of the habits of the unprepared pedagogue and
prepared herself through study and reflection. This process is illustrated in Figure 2.
Once again, Chen was learning almost entirely through experimentation. She lacked formal training and there were no teachers or mentors in her current life who could reassure her that her experiments were successful. However, she was proud of her own bravery and felt a certain amount of confidence that she was on the right track. The changes Chen made to her instruction, particularly in this third phase, may seem disparate and far-flung, but I found them to be accurately summarized by the concept of developmentally appropriate conversation. With these changes, the children were finally brought into the active process of co-constructioning norms for conversation. Now that they were highly involved in conversation, the children only required classroom management twenty-six times over the course of five weeks compared to thirty-eight times in the first two weeks and twenty-five times in the third and fourth weeks alone.

By the fifth week in the study, Chen shared with me her desire to learn more strategies for engaging children in conversation. At this point in our discussions, I shared with Chen that I had observed her using two research-based conversation strategies. I had noticed her revoicing (Chapin & O’Connor, 2007) and recasting (Temple & Doerr, 2012). In revoicing, the teacher repeated a child’s contribution so that the rest of the class could consider it and respond to it. In recasting, the teacher repeated a child’s contribution, altering it slightly to make it more technically correct. As in the case of revoicing, the purpose of recasting was to offer a child’s contribution for explicit consideration by the child’s peers.

When I shared this observation with Chen, she immediately asked to see some of this research about conversation strategies. I shared with her the study on talk moves by
Chapin and O’Connor (2007). Chen studied this article carefully in her own downtime. In the subsequent weeks she began using the other conversation strategies described in the study during her math lessons. During weeks five through twelve, Chen practiced using these and other strategies that happened to be research-based, but she also developed many of her own. While I only shared Chapin and O’Connor’s research with her, she naturally developed some of the strategies outlined by Temple and Doerr (2012).

During this phase of the study, Chen stated in interviews that she felt that something “clicked” for her and she was able to engage children more completely in conversation. Her style of conversation was now thoroughly developmentally appropriate. This period of the study was an exciting one for Chen. For most of her career, she had stuck to fairly traditional curricula and had usually engaged children only superficially in verbal interactions. Up until now, her style of verbal interactions was teacher-directed and not developmentally appropriate. She shared with me why this was the case:

When I first started out, like right out of my master’s degree program, I tried hard to talk to my students, who were in kindergarten. But I didn’t have any of these ‘talk moves’ as these guys call them. You could say, like, I didn’t have any tools in my toolkit. So that when I tried to chat with the children, the conversations would always go in directions that I . . . that I didn’t intend. I didn’t think they were very instructive conversations. Maybe I was wrong. That was when I was younger. Then I just developed my own habits I guess. I didn’t let the children talk.
As a result of these initial negative experiences with child-teacher conversations, Chen spent most of her career sticking close to traditional curricula and engaging children in fairly superficial verbal interactions. She thought she was doing the right thing by focusing the children on the material her administrators had asked her to share with them. She did not think that active conversation was the very thing that would facilitate children in concept formation (Vygotsky, 1934/1978).

Because of her discovery of these conversation strategies, which I describe in detail in a subsequent section, the children were able to co-construct sociomathematical norms with the teacher. No longer were rules simply being imposed upon the children. Chen, the central participant, brought her students, who were peripheral participants, into more central participation (Lave & Wenger, 1999; Yackel & Cobb, 1996). The conversation strategies enabled children to participate more centrally as they co-constructed sociomathematical norms. Increased central participation was critical for the co-construction of sociomathematical norms. The closer to central participation a child moved, the more privileged he or she was in changing the norms that governed participation.

In the following section I describe the sociomathematical norms that guided children’s conversation during Chen’s math lessons. Then I describe the norms for Chen’s role specifically in these conversations. I also describe how Chen and the children co-constructed these norms and how these norms guided subsequent conversation.
Norms for Children’s Premath and Mathematical Conversations

In this third phase of the study, conversation was far more developmentally appropriate than in the previous two phases. Chen allowed and even expected children to verbalize their reasoning. The children communicated directly with their peers. They also talked through disagreements. They asked each other questions. Finally, they cleared up each other’s premath and mathematical misconceptions.

Verbalizing reasoning. Many of the rules in the second phase (new rules for conversation) created opportunities for children to verbalize their reasoning. Allowing the children to speak for longer times, allowing them to speak without raising their hands, and allowing children to speak when they requested to all made it more likely that children would verbalize their reasoning. Further, when the teacher explored the students’ reasoning, she was fundamentally modeling for them how to verbalize their reasoning. By the third phase, it was apparent that the children internalized this norm Chen had modeled. Chen verbalized reasoning for the children in the second phase, but by the third phase the children were co-constructioning this norm so that they sometimes verbalized their reasoning without prompting, and in their own unique ways.

The children’s internalization of this norm was apparent in week seven when they worked in the small group setting to build a structure out of marshmallows and dried spaghetti noodles. Chen worked with each group as the other children played at centers while waiting their turn. As was typical for her in the small group setting, Chen spoke minimally to encourage the children to chat with each other. Chen generally used more conversation strategies in the whole group setting. In the small group setting, she let the
children take the lead to a radical degree. Chen reminded the children to verbalize their reasoning, but she did not dictate how they were to do so. In order to work together to build the structure, the children had to be creative in verbalizing their reasoning and their plans to each other. In one instance, Hannah, Beth, and Logan had constructed a cube-like structure. The structure was beginning to fall apart. Beth talked to her team members to notify them that she was holding it up so it would not fall. Hannah added that she was holding onto it too and that she planned to add another stick to buttress the structure up on the side toward which it was falling. Throughout weeks five through twelve, there were occasions when the children had to verbalize their reasoning to each other in order to solve problems, sometimes with manipulatives and sometimes without. In this case, the children were verbalizing their reasoning as they worked to build a structure.

“If we add one more stick, it will make the house strong,” Hannah reasoned. “I’m going to add a stick here. Will you hold it, Beth?” Then Hannah turned to Logan, “Can I have that stick?” Logan said no. Hannah explained to him that if they did not use it, the house would fall. Hannah finally broke off a small piece herself and attached it to the structure.

However, the structure kept falling. Hannah continued to verbalize her reasoning: “I have an idea! We should add a piece to the inside. A diagonal piece. That will make it steady.” Chen pointed out to the other children that diagonal was a very good math word to know. Hannah broke off a piece of spaghetti noodle and attached it inside of the structure. “I think it can stand now. Let go!” Hannah said.
Beth next verbalized her reasoning: “It needs a piece inside. It’s not strong inside. That’s why it’s falling.” Once the noodle was attached, the children let go of the structure and it stood for five seconds.

Chen informed them that they won the game since the structure stood for five seconds. There were many instances in the third phase in which children verbalized their reasoning to each other or to the teacher. However, there were also negative cases in which Chen prompted children to verbalize their reasoning and they still did not do so. More often than not, children solved problems with manipulatives without verbalizing their reasoning in doing so. This norm was thus a work in progress. Chen shared with me the thought that if she had enculturated the children into conversation from the beginning of the year instead of just during my study, they would certainly have gone farther by the end of the year in verbalizing their reasoning. This study took place during the last three months of the school year. Hence, many of the teacher’s and students’ habits for math lessons were ingrained. For this reason, the changes they were able to make to their habits were that much more remarkable. Giving children freedom to make personal connections to the pedagogical material was one such change.

Making personal connections. In phase one of the study, Chen did not allow personal connections. In phase two, she changed her practice significantly by avidly encouraging personal connections. I would argue that this was the single most developmentally appropriate change Chen made to her conversation practices. Preschool-age children need to make personal connections to instruction in order to truly access that instruction (Copple & Bredekamp, 2010). In phase three, she went a step
further and talked explicitly about what it meant to make a personal connection. She wanted children to think explicitly about how their personal connections could help them deeply internalize mathematical knowledge and reasoning. In the third phase, Chen often reminded the children that when she taught them about a new math idea, it would be good for them to think back in their memory and try to remember a time they used that idea at home or when they were with their families. Thus, she did not just allow children to make personal connections; she explicitly encouraged them to.

During a lesson in the sixth week, Chen told the children, “So while we talk about money, I want you to think about times you used money or your mommy or daddy used money. That’s called making a personal connection.” Chen had the children repeat the words personal connection. After Chen made this statement, she engaged the children in an extended, open-ended conversation about their personal experiences with money.

During this conversation they talked about how much was a little bit of money and how much was a lot of money. Chen offered the example that the children got a little bit of money from the tooth fairy. Olivia stated that she had a personal connection to share. Chen praised her for remembering the phrase, personal connection. Olivia then stated that she had received a hundred dollars from the tooth fairy. Chen laughed and then asked the children if one hundred dollars was a little bit or a lot of money. Most of the children said that it was a lot of money. A few children guessed that it was a little. Chen said that she was glad Olivia had made this personal connection because it would help them to learn about math. This was a dramatic change from Chen’s practice in the first phase in which she discouraged personal connections. In the third phase, it was
typical of her to look forward to them and to incorporate them into instruction. Chen stated now that Olivia’s contribution would help the children think about how much money was a little and how much was a lot. She asked the other children to make their own personal connections by telling her how much the tooth fairy gave to them. Chen went around the circle asking each child how much he or she got from the tooth fairy. Some children said one dollar, some two. Others said five or ten. Chen noted that the tooth fairy gave different amounts in different houses, but she always gave a pretty small amount.

Hannah said that she also had a personal connection to make. Chen permitted her to share. Hannah said that when she was in New York, she had lost her tooth. There in New York, the tooth fairy gave her two dollars. The next day, she used the two dollars to buy a hot dog all by herself. Chen listened to Hannah’s whole story, allowing her to elaborate upon other things she did during that vacation. Chen reasoned later during our interview that it was best for Hannah’s learning for her to share all the personal information that she wanted to. This was typical of Chen in the third phase. She was, again, the courageous experimenter in her math lessons. She kept in mind what she had heard in her master’s degree program as well as what she heard in the in-service. She knew conversation was important, and she had figured out on her own that conversation for preschoolers meant lots of personal connections. At the end of Hannah’s contribution, Chen called the children’s attention to the fact that two dollars was enough to buy just one hot dog. She asked the children if that was a lot or a little. Most of the children now answered that it was a little. Chen described other amounts of money as well as how
much the children would be able to buy with them. She added that for a hundred dollars, their whole family could go to a nice restaurant and eat a lot. After giving this information, she asked Olivia if a hundred dollars was a little or a lot. Olivia admitted that it was a lot after all.

This example conversation illustrates the way the children co-constructed with Chen the norm of making personal connections to mathematical content in the third phase of the study. In this and many other examples, they complied with Chen’s request that they make personal connections, but they did so in their own unique ways. Another common occurrence was that personal connections led to mathematically pertinent discussions, such as above where the children talked about what constituted a little or a lot of money. Making these personal connections guided the children to talk with each other about the mathematical topics being discussed in the classroom.

Children communicating directly with peers. In phase two, Chen prompted students to talk directly with their peers. This was another means of making premath and mathematical conversations more developmentally appropriate. The children practiced doing so, at Chen’s encouragement, during the third and fourth weeks. By the fifth through twelfth lessons, the children were accustomed to conversing with each other and no longer required prompting to speak directly to one another. They told stories to their peers, they verbalized their reasoning in solving problems, and they argued with one another about the best way to approach a problem.

In the seventh week, during the small group activity in which the children made a structure out of marshmallows and spaghetti sticks, Elodie and Aarav engaged in
conversation with each other. Even Aarav, a student in the linguistic minority, had grown accustomed to conversing with his peers, at least in small group time. In this instance, Aarav and Elodie had created a rectangular structure. Aarav, who was usually quiet or even silent, was thoroughly engaged during this activity, manually and verbally. As Elodie held the structure up, Aarav said that he wanted to connect a corner marshmallow with the one holding up the roof. He found that Elodie’s hands were in his way. Aarav told Elodie that she was not listening to his words. Chen had instructed the children to speak to one another using this verbiage. Chen now praised Aarav for being assertive with Elodie. Elodie responded directly to Aarav, saying that if she moved her hands, the structure would fall. Aarav asked her to hold it a different way so that he could insert the stick. Elodie changed her hold on the structure and Aarav placed the stick in the position he had described. The structure fell afterwards, but Chen praised the children for speaking directly to one another during this problem-solving activity. Throughout the rest of the study, I observed children speaking directly with one another. Chen continued to reinforce this behavior with praise. However, she did not dictate to the children how they were to speak to each other. Rather, the children had to figure out how to do so on their own, and they often solved problems effectively by conversing with each other.

Children asking each other questions. Along with teaching children to speak directly to their peers, Chen also encouraged children to ask each other questions. She wanted her students to grow accustomed to communicating with each other and not just with her. Just as the children co-constructed the norm of speaking directly to one another,
they also co-constructed the norm of asking each other questions. Children asked each other questions in all three settings: whole group, small group, and pairs. They asked each other pre-mathematical as well as mathematical questions. They asked each other questions when they wanted to elicit ideas from each other, when they were trying to solve a problem they did not know the answer to, or rhetorically when they were acting as the teacher to their peers. In the example below, Emily and Elodie asked each other a series of questions to elicit ideas from each other as well as to clarify what mathematical processes they were using. It is notable that although the children already engaged in conversation with each other most often during pair work in the second phase of the study as well, they engaged in more mathematically productive talk together in this setting in the third phase. It made sense that they did so because they had gained practice engaging in pre-math or mathematical conversations during whole group time.

In week twelve, Emily and Elodie were playing as a pair with transportation counters. Emily asked Elodie what she wanted to do. Elodie said she wanted to make a pattern. Emily concurred that this activity would please Miss Chen. Elodie then asked Emily what kind of pattern she wanted to make. Emily did not answer. She simply began to make a pattern. Elodie began making her own pattern using the transportation counters as well. Emily’s pattern was red, orange, yellow, yellow. Elodie’s pattern was red, blue, green, blue. Emily asked Elodie if she was really making a pattern and not just a design. Elodie argued that it was indeed a pattern because she was repeating the whole thing. The girls went back and forth asking each other questions and making comments on each other’s work as they completed their patterns.
This episode can be contrasted with another episode in phase two when Logan and Nancy made patterns with teddy bear counters. The two children sat side by side at the table making their patterns. In spite of their close proximity, the children only talked with each other when they were arguing over who had more counters than the other. Although they were sitting together engaged in the same activity, they did not ask each other questions or make comments on each other’s work. This case illustrates that not all of the children had appropriated the sociomathematical norm of asking one another questions. Once again, if Chen had enculturated the children into this practice earlier in the year, the children would have been more active in co-constructioning this norm.

Clearing up one another’s misconceptions. As the children internalized the norm of speaking directly to each other, they began to teach each other and to clear up one another’s misconceptions. They did so in all the settings and in many ways. The children had many misconception about the various topics that Chen taught about—the teen numbers, time, money, light and shadow, how to build, and how to count. In phase three, the children began to actively clear up one another’s misconceptions. They had the opportunity to do so as a result of Chen’s ongoing pedagogical experiments. Chen encouraged the children to teach each other as yet another way for them to take ownership over their learning. The freedom the children enjoyed to clear up one another’s misconceptions led the children to be engaged and to internalize mathematical practices.

In week five, during Chen’s first lesson on money, Chen asked the children why the people in the store sometimes give you some of your money back after you buy
something. Logan called out that they do that because they want you to have more money. Chen did not tell him he was wrong. She preferred for the children to clear up one another’s misconceptions. She had decided that the students would be more likely to remember one another’s contributions than her own.

Chen turned to the other children and asked if anyone else had an answer to her question. Hannah spoke up and told Chen that what she was talking about was called change. Chen scaffolded the conversation by revoicing Logan’s contribution for Hannah. She asked Hannah what she thought about Logan’s contribution. Hannah told Logan directly that what he said was not right. She said that change is the extra money that the people in the store give you back. Chen asked the other children what they thought. Mahesh stated that he agreed with Hannah. Mahesh said that the people at the store were not allowed to keep your money if you gave them too much. The conversation continued at length. The children took out the Unifix cubes to demonstrate to Logan how the concept of change worked. After hearing from his peers, Logan conceded that change was the money the storekeeper gives you when you hand him or her too much money.

In the above example, Chen’s students co-constructed the norm of clearing up one another’s misconceptions. They made this norm their own by choosing how to speak to their peers. They did so with only minimal scaffolding on Chen’s part. In an earlier phase of the study, the children may have become irate over disagreements. The children’s adoption of the norm of talking through disagreements also helped support them in clearing up one another’s misconceptions. In this and other ways, the norms were interconnected. I observed that children increasingly cleared up one another’s
misconceptions as the study went on. As the children adopted this norm, they took
ownership over their learning and internalized the material more readily. They did so in
the whole group, small group, and pair work settings.

Norms forTeacher’s Role in Supporting Pre-math and Mathematical Conversations

In phase two of the study, Chen enculturated the children into holding pre-math
and mathematical conversations via her explicit and implicit conversation rules. As
described above, in phase three, the children had internalized many of these rules in order
to engage in more conversation with less prompting by Chen. In phase three, the children
began co-constructioning sociomathematical norms with the teacher and with each other.
Also starting in phase three, Chen began supporting conversation in other important ways.
She supported conversations with manipulatives, with visual aids, and with kinesthetic
activity. She began holding children’s comments for later, so that she was able to
validate more of the children’s contributions instead of dismissing them. She also began
using conversation strategies, which enculturated children into certain conversation
behaviors. I have described these conversation strategies at length in a section above. In
the following section, I describe how Chen used manipulatives, visual aids, and
kinesthetic activity to support conversation.

Supporting conversation with manipulatives. Chen used manipulatives in the first
week of the study, during the lesson on teen numbers. However, she did not use them to
support conversation. Rather, she used them to help children answer questions correctly.
Rather than listening to the children’s contributions as they used these manipulatives, she
talked them through steps to take to arrange the manipulatives. In most of her lessons in
phase one and phase two, Chen did not use manipulatives at all. Looking over the transcripts for these lessons, Chen noticed that children often reached out for physical objects in their environment to play with. In these instances, Chen reprimanded the children and asked them to return to their spots. Reading about these incidents in the transcripts, Chen realized that she should make manipulatives more central to lessons so that children could enjoy a tactile experience. This was another way in which Chen made her practice more developmentally appropriate. Chen remembered a quote from the in-service that we should teach children in the way that they learn. Since they longed to touch objects during her lessons, she made a plan to use manipulatives so that they could touch objects. She hoped to incorporate manipulatives in a way that would support premath and mathematical talk. The use of manipulatives helped children further internalize the sociomathematical norms described above.

In week five, Chen allowed the children to solve a problem using Unifix cubes. In week six, the children played with trays of coins. In week seven, the children made structures using marshmallows and spaghetti noodles. In week eight, they built structures out of blocks and magnetic tiles and measured the shadows of these structures. In week nine, they played a “making 10” game with the colored circles in a cup. In week ten, they played another “making 10” game in which they filled egg cartons with Unifix cubes to make ten. In week eleven, they played a board game. In week twelve, they rotated in pairs through manipulative centers. Thus, in every lesson during the sociomathematical norm phase of the study, Chen made sure the children engaged with hands-on materials to support their learning and talking about mathematical concepts.
These lessons had a number of elements in common. In each, Chen gave the children a rough outline of how the manipulatives were to be used. After giving the children some direction, Chen stepped back and allowed them to take the lead in how to use these manipulatives to solve problems or for free play. Chen stepped in at opportune times to introduce mathematical ideas or language. Only in some instances in phase three did Chen dictate how the children were to use the manipulatives. In these instances, she did so after the children had struggled independently with a problem without being able to resolve it. Occasionally and purposefully, Chen still walked the children through how to use a manipulative to solve a specific problem, but she only did so after the children had struggled through conversation to come to a consensus about a mathematical idea. In the example below, a child introduced the idea of using Unifix cubes to resolve a disagreement. Then, Chen walked the children through a sample problem and solution that helped them conclude their debate.

This illustrative incident occurred in week five. In this lesson, Chen engaged the children in a productive conversation about money: what it was, how it was used, and where it came from. Every child spoke during this conversation. One factor that made this conversation a success was Chen’s skillful use of Unifix cubes. Over the course of this conversation, Logan verbalized the misconception that people in stores give people their money back because the people in stores want the people to buy more things. Chen allowed other children to disagree with him. As I have described above, Hannah and Mahesh pointed out that this extra money was called change.
Logan was not immediately convinced. Chen tried to demonstrate with her fingers how shop keepers determined how much change to give back to customers. Logan was still skeptical. It was possible he did not understand the problem Chen demonstrated using her fingers. Hannah suggested that they use Unifix cubes to show Logan what they were talking about. The children decided to demonstrate the idea of change through a make-believe scenario in which Logan was the shopkeeper and Forrest was his customer. Chen suggested that Logan start out with ten Unifix cubes and buy something for five dollars. Hannah told Logan to choose something from one of the centers. Logan chose a firefighter’s hat.

When Logan brought the hat back to the circle, Hannah told him to pretend to be a shopkeeper. Chen asked the children who wanted to be the customer. Forrest got up and said that he would be the shopkeeper. Chen asked Logan to count out ten Unifix cubes to represent Forrest’s money. She told Logan that the ten Unifix cubes would represent ten dollars. Logan made a bar of ten and gave it to Forrest. The make-believe commenced. Forrest pretended to be a customer and gave Logan the bar of ten. So, Chen asked Logan, if he gave you ten and the hat only costs five dollars, what should you do? Logan stated that he had to give Forrest back the extra. Logan broke off a bar of five cubes and gave it to Forrest. Chen asked Logan once more why shopkeepers give people their money back. Logan conceded that it was extra money. Where fingers failed to help Logan to talk out the problem, the use of manipulatives was successful. In the above example, the children’s idea to use a manipulative led to an imaginative mathematical conversation. From previous experience in Chen’s classroom, Forrest
understood how to use the cubes to represent ten dollars. Logan knew to break off a five bar to represent five dollars. Engaging directly with the Unifix cubes, and the resulting conversation he had with his peers, helped convince Logan of the mathematical fact that shopkeepers give customers change. In this and other subsequent lessons, the use of manipulatives aided children in internalizing the sociomathematical norms of speaking directly to one another and of clearing up one another’s misconceptions. Now, I will turn to discussing ways in which Chen supported the ongoing mathematical conversation in her classroom using visual aids.

Supporting conversation with visual aids. In addition to using manipulatives to support conversations, Chen used and taught the children how to use visual aids. Chen used books, game sheets, and the hundreds chart as visual aids. These visual aids helped Chen persuade the children of mathematical truths. They also helped the children persuade each other. The visual aids helped the children to think about mathematical ideas and to talk about them. More than anything, the visual aids supported speakers as they strove to convince each other of their mathematical arguments. For example, the whole group used the hundreds chart, which was a chart hanging on the wall that showed numbers one through one hundred organized in rows and columns of ten. Either the children or Chen referred to the hundreds chart at some point during almost every lesson.

In the sixth week, during an open-ended conversation about money, Olivia claimed that the tooth fairy had given her 100 dollars. Chen laughed, some of the children exclaimed that they wished they got 100 dollars from the tooth fairy. Still others shouted out that the tooth fairy could not give you 100 dollars. Olivia held her ground,
insisting that she had received 100 dollars from the tooth fairy. At one point in the conversation that followed, Chen suggested that they use the hundreds chart to think about big and small numbers. Chen asked who could act as the teacher and teach them about big and small numbers on the hundreds chart. Chen chose Hannah because she wanted to give this gifted student a chance to show off what she knew.

Hannah got up and went to the hundreds chart. Hannah took the pointer and said, “Look, Olivia. Up here is number one. That’s very little. When you move down the chart, it gets bigger and bigger. One hundred is at the very bottom. It’s the last number.”

Chen asked the children what it meant that 100 was the last number on the chart. Mahesh spoke directly to Olivia that it meant 100 was the biggest number on the chart. In this example, although the teacher initiated the use of the visual aid, the two gifted students knew just how to use it to teach her peers about big and small numbers.

However, it was clear in other scenarios that Chen’s support was still needed to help the children co-construct the use of visual aids to internalize other sociomathematical norms such as speaking directly to each other, talking through disagreements, and clearing up one another’s misconceptions.

Supporting conversation with kinesthetic activity. Chen frequently made use of manipulatives and visual aids to help support children’s conversation and co-construction of sociomathematical norms. To a lesser extent, she made use of kinesthetic activity. Although Chen used kinesthetic activity in fewer lessons than she used visual aids, her use of kinesthetic activity reflected her willingness to experiment and teach the children the way that they learned. Chen had noticed that the children felt constricted sitting in
one spot for too long, and that they longed for physical activity during her math lessons. Chen drew upon her own resources to change her instruction to make it more developmentally appropriate. In one lesson in particular, I noticed that the use of movement helped support conversation. In lesson six, Chen taught the children a song and dance about money. The lyrics of the song were, “A penny’s worth one, a nickel’s worth five, a dime’s worth ten, and a quarter twenty-five. Hey, honey bunny. I know my money money.”

Chen and the children sung the song and rehearsed the dance several times before the children were broken up into pairs to play a coin matching game. To play the game, Chen gave each pair a laminated sheet of paper and a tray of coins. On the game sheet was a picture of a piggy bank filled with circles. Inside the circles were numerical values of one cent, five cents, ten cents, and twenty-five cents. The song and dance served as a mnemonic device while the children played this game. Chen had found this song on Pinterest as part of her efforts to engage children in kinesthetic activity. As the children took turns matching the coins to the circles on the laminated page, they got up from their seats to sing the song and go through the dance. Doing so successfully helped them remember the values of the coins.

In many instances during this game, the pairs of children disagreed on the values of the coins. The song and dance helped speakers convince each other of the value of each coin. Thus, the norm of kinesthetic activity helped these children internalize the norms of conversing directly with one another, that of talking through a disagreement, and that of clearing up one another’s misconceptions.
Holding a student’s comment or question. Chen showed improvement in her pedagogical skills during the later weeks when she was able to hold a student’s comment or question temporarily before returning to it during a lesson. In phase one of the study, Chen often asked children to hold onto their comment or question for later when she saw this comment or question as veering off-topic. As I noted above, she never came back to these comments or questions as she said she would. In phase two, Chen changed her practice radically to follow children’s lead in conversation. She tried to follow children’s lead even when they were changing topics. In phase three, however, she engaged frequently in a new practice that helped her better manage conversations. When a child contributed in an off-topic way, Chen first acknowledged the contribution and then asked the child to hold onto it for later. In this phase, Chen actually came back to these contributions when it was possible. Thus, Chen did not let children stop conversation flow with off-topic contributions, but she did create the space for a change in topic later on in the lesson. For example, in week five, during the conversation about the nature of change, Emily interrupted the conversation to say, “Some people are rich. They have a lot of money. And and and some people are poor and they don’t have a lot of money. Why come some people are poor?”

Instead of stopping Emily from speaking, Chen validated her contribution by summarizing her change of topic for the children. Chen noted that Emily wanted to talk about a little money versus a lot, and that this was a good math topic. Chen also made it clear that Emily’s statement represented a change of topic. She asked Emily if they could
finish their conversation about change before they talked about why some people are rich and some are poor. Emily agreed.

Unlike in the first phase of the study, in this lesson that took place in phase three, Chen was true to her word and came back to Emily’s topic as soon as they had all reached an agreement about what change was and why people gave and received it. The group went on to have just as lengthy a conversation about how people earn money, and why some people have more money than others. As I describe below, a child actually introduced this norm of holding a comment or question for later. By co-constructioning the norm of holding onto a topic for later, Chen enculturated the children into the practice of following a train of thought to its logical conclusion before changing topics. She also arguably enculturated them into the positive evaluation of all contributions by giving them attention at the right time. By letting all the children express themselves, Chen ensured that conversations were developmentally appropriate.

In a classroom micro-community, norms for thinking and speaking are co-constructed by all members of this community in myriad imperceptible ways (Lave & Wenger, 1999). However, there were four means by which norms were visibly constructed. Sometimes Chen planned ahead for the construction of these norms. At other times, children introduced their own norms and Chen embraced these norms. In these cases, she was planning on her toes. At other times, a norm was constructed when Chen disciplined a child for behaving in ways contrary to a norm. This was not a method she planned for during her own time. Rather, the need for discipline arose spontaneously during math lessons. Here, again, Chen was planning on her toes. She was no longer a
completely unprepared pedagogue, but she was still learning things about her own style of conversation. Finally, Chen planned ahead and made sure that several norms were co-constructed when the she relaxed old rules. This planning ahead of time and planning on her toes reflected Chen’s emerging identity as a pedagogical experimenter. Chen was making up for her lack of preparation by experimenting and learning from these experiments. Table 2 presents examples of the various catalysts for norm construction. Although there were surely myriad other, subtler ways that norms were constructed, there were four highly tangible means by which norms were constructed.

Table 2

*Catalysts for Constructing Norms*

<table>
<thead>
<tr>
<th>Catalyst</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher explicitly introduced a norm.</td>
<td>Chen told the children she wanted them to talk directly to one another, to talk through disagreements, and to verbalize their reasoning.</td>
</tr>
<tr>
<td>A child introduced a norm.</td>
<td>Olivia interrupted the conversation to ask a slightly off-topic question. Mahesh suggested that they hold the question for later. The teacher confirmed that they would simply put this child’s contribution on hold.</td>
</tr>
<tr>
<td>Punishment led to a norm.</td>
<td>Emily was working on a math game with a partner. She was not verbalizing her reasoning as Chen was requesting she do. Instead, she was lying down on the carpet. Chen threatened to send her to the quiet tent. Emily sat up and began to contribute, verbalizing her reasoning.</td>
</tr>
<tr>
<td>Teacher relaxed old rules.</td>
<td>Chen orchestrated a conversation about how to make up the number ten in a math game. She allowed the children to speak without raising their hands. In so doing, she introduced the norm of following the children’s lead in conversation.</td>
</tr>
</tbody>
</table>
How Norms Guided Conversation

Just as the teacher and children co-constructed sociomathematical norms in countless subtle ways, so it is certain that in the process of their construction, these norms guided conversation in immeasurable, barely perceptible ways (Lave & Wenger, 1999). However, as I reviewed the data, four broad ways that sociomathematical norms guided conversation emerged. The co-construction of norms led to increased practice in conversing with the teacher. Children also practiced talking more often with each other. Students in the linguistic minority were more often included in conversation and thus gained more practice conversing. Finally, the children shared responsibility for conversation more often.

Increased practice conversing with the teacher. In the third phase of the study, the phase in which the children co-constructed sociomathematical norms with the teacher, the children increasingly practiced conversation with the teacher. The children made this transition easily. The children who, during the first phase of the study would lay their heads in Chen’s lap now happily conversed with her. They were visibly happy and eager to share their thoughts and ideas with Chen. Chen, for her part, encouraged them to converse in increasingly complex ways.

In week eight of the study, Chen taught a lesson in the small group setting. In this lesson, Chen had the children build structures with blocks. Then she shone a light on the structure and had the children measure the length of the shadows created. Chen conversed less with the children in the small group setting than she did in the whole
group setting, and yet even in the small group setting, the children had opportunities to practice conversing with Chen.

The first group that Chen called to work on the structure together included Olivia and Emily. The group conversed among themselves as they built. Then Emily told Chen that she was going to get something that would be really good for their building. Emily brought back the translucent magnetic tiles. Chen asked Emily what these would do. Emily said that they would glow and make colors when they shone light on them, and that they could measure the part that glowed. Chen clarified with Emily, asking her if the light would make colors. Emily confirmed that this was her guess. Chen asked her why she thought this would be the case. Emily stated that the magnetic tiles were made of different stuff, so that their shadows would probably look different. Emily added that the shadow would still be long when they measured it. In this example, Emily’s verbalization of her reasoning led her to engage in more extended conversation with Chen. The rule of speaking for extended times and the norm of speaking without raising hands also made this open-ended conversation possible. Finally, the sociomathematical norm of supporting conversation with the use of manipulatives also enabled these students to practice conversation with the teacher. As the study progressed, the children had opportunities to converse with the teacher even in the small group setting.

Children practiced talking more often with each other. During the same scene described above, Chen asked Olivia and Emily what they should use to measure. Emily said she thought it would be better to use tape measure. Olivia answered that it would be the same no matter what they used to measure it. Chen asked Olivia to explain to Emily
why it would be the same. Olivia said that the ruler and the tape measure both had the same numbers on them.

Here, two norms led to children’s increased practice conversing with each other. The children did not have to raise their hands to speak. They were also encouraged to speak directly to each other. The group went on to turn the lights off, to shine the flashlight on the structure, and to measure the shadows. Olivia and Emily took turns measuring the shadow with the ruler and the tape measure, conversing with each other about how the results were the same.

Linguistic minority students gained opportunities to converse with the teacher. As norms progressively came to guide conversation, students in the linguistic minority gained more opportunities to engage in conversation with the teacher. Most often, Chen had to initiate conversation with these children, although they volunteered their own contributions at times. As noted previously, the students in the linguistic minority were most likely to volunteer contributions when working in pairs or small groups. Aarav and Forrest were more likely to speak in the small group setting than in the whole group setting. In the same lesson about light and shadow, Chen called Aarav, Nancy, and Elodie to work together on a structure. Chen decided to keep the magnetic tiles Emily had brought into the activity. When Chen shone the light on the structure, the structure cast a long shadow on the table. Chen moved the light closer and then further away from the structure, and the children measured the shadow as it shrank and then grew.

Chen asked the children why they thought that the magnetic tiles’ shadow was different than that of the wood blocks. Aarav spoke up and said that the light was
“showing” through the tiles but not through the blocks. Chen revoiced his answer for the other children. She praised him for verbalizing his reasoning. But Chen wanted to press the children further in verbalizing their reasoning. Chen asked the children to describe the shadows very carefully and to tell her what was different. Aarav again spoke, saying that part of the shadow was a triangle and it was blue in the middle. Chen asked him why that part was blue and the rest was black. Aarav pointed to the magnetic tile and said that it was because this was blue. Chen pointed to a blue wooden block and said that it was blue too, but its shadow was black. She asked again, why? Aarav spoke again, saying that the wooden block and the magnetic tile were not the same. He said that the magnetic tile was like glass. The other children agreed, saying that the magnetic tile was not dark enough to make a black shadow.

In this example, as in the one above, the sociomathematical norm of verbalizing one’s reasoning led the children to engage in deeper, extended conversation with the teacher. In this case, a student in the linguistic minority, Aarav, led the children’s discussion with the teacher. Aarav did not typically lead conversations. However, this special example demonstrated how far he came in his ability to join in conversation. Chen speculated about Aarav’s increased participation in this conversation, guessing that it was in part due to the fact that they were conversing in the small group setting. She also felt strongly that her own efforts to normalize conversation during math lessons emboldened Aarav to speak. The data did show that the more that Chen brought Aarav into the conversation, the more he came out of his shell to volunteer contributions at other times, especially in the small group or pair work settings.
Children shared responsibility for conversation with the teacher. The children shared responsibility for conversation with the teacher. Very often, instead of simply giving information to the children, Chen elicited the information from the children. The children, for their part, were happy to take on responsibility for the conversation and provide each other with the needed information. The children shared responsibility for conversation when called on and often when they were not called on. They shared responsibility for conversing with Chen as well as with each other.

In a lesson during week six, Chen wanted to review some facts about money. Chen asked the children what money was used for. Beth gave a lengthy answer, the gist of which was that money was for buying things: “You give it to the person at the store,” she said, “and they give you change. You get to keep the change and you also keep the thing you bought.”

Next, Chen asked them what they learned from participating in the bake sale. I thought Chen would have to ask a more specific question to elicit answers from the children, but then Olivia shared without further prompting all that she learned. She said that one way to make money is to sell things. She said that at the bake sale, people came up to their table and gave them money. Then they gave the people their change and the cookies they had bought. Olivia continued on at length about what she had learned from the bake sale, and her peers spoke up to add to or clarify her statements.

I often observed that, in the third phase, the children were ready with appropriate answers when Chen asked an open-ended question. As noted above, it had been difficult for Chen to transition to a more conversation-rich style of instruction, but it was easy for
the children. It was clear that conversation-rich instruction was appropriate for their phase of development. With practice in the second and third phases, the children learned to actively co-construct sociomathematical norms with Chen.

The children who participated least in the co-construction of sociomathematical norms were the two students in the linguistic minority: Aarav and Forrest. Although they did participate in some of the conversations, they did not participate as often or as extensively as the other children. This was the case in spite of Chen’s best efforts to include them. Because of this low level of participation, these students also participated less in the co-construction of sociomathematical norms. The result was an emerging classroom culture that increasingly reflected the other students’ cultures and not the cultures of Aarav and Forrest. In this discussion of Chen’s success in engaging children in the co-construction of norms, it is important to acknowledge that not all of the children participated equally.

*Figure 3* illustrates the recursive movement between the teacher’s and children’s behaviors which moved the group culture from initial rules for verbal interaction to new rules for conversation and then sociomathematical norms. The teacher’s behaviors, as described in the sections above, led to certain behaviors in the children, which then inspired the teacher to change her practice. The figure illustrates the fact that the teacher and students were indeed co-constructioning norms throughout the study although this norm construction was only overt in the final phase of the study.
Figure 3. Recursive movement between the teacher and children’s behaviors. Initial contributions of children and teacher moved communication from simple interactions to new rules for conversation. These initial contributions as well as later ones moved communication from new rules for communication to sociomathematical norms.
Summary

My findings focused on how a teacher in one preschool classroom co-constructed sociomathematical norms with her 10 students. I have related how the teacher used conversation strategies and other strategies to open the floor for more developmentally appropriate conversation. I used Lopez and Allal’s (2007) expanded interpretation of Cobb and colleagues’ (Cobb & Bowers, 1999; Cobb et al., 1997; Cobb et al., 2001; Cobb & Whitenack, 1996; McClain & Cobb, 2001; Yackel & Cobb, 1996)) conception of sociomathematical norms to analyze the cooperative style in which the teacher and students co-constructed norms for thinking and speaking mathematically. The teacher went on a journey in which she progressively changed her practice to incorporate more opportunities for conversation into her math lessons. She also enculturated the students into specific practices appropriate for premath and mathematical conversation.

In analyzing the data, I found that the teacher and children changed their conversation practices over the course of three successive phases. In the first phase, the teacher imposed rules for initial verbal interactions. True conversation did not take place during this phase. The teacher’s reflection on the transcriptions of this first phase led her to engage the children in more developmentally appropriate conversation. In the second phase, the teacher established new rules that allowed true conversation to take place. These conversations were increasingly developmentally appropriate. In the third and final phase, the teacher encouraged the children to co-construct sociomathematical norms that guided conversations going forward. I found that as the teacher engaged children in co-constructioning norms, children became more engaged in premath and mathematical
conversations, they took on more responsibility for conversation, they remembered a great deal more of the lesson contents than they had in the first, and children conversed more equitably. The result was a more engaging atmosphere in which the students conversed with each other and with the teacher alike. *Figure 4* is an integrated model depicting the co-construction of sociomathematical norms.

*Figure 4.* Integrated model. Obstacles stood in the way of productive conversation and thus of co-construction of sociomathematical norms. The teacher and students overcame obstacles to co-construct sociomathematical norms.

The outer ring of the model represents the obstacles presented by the teacher as well as by the students. The boundary separating the obstacles from the settings is shown in jigsaw pattern to show contention. The efforts of the teacher and students to overcome obstacles to conversation, and thus to co-construct sociomathematical norms, met with
contention at this boundary. However, the teacher and students were successful in crossing over this line in order to co-construct sociomathematical norms.
CHAPTER 5
DISCUSSION, CONCLUSIONS, AND IMPLICATIONS

I conducted this qualitative study to examine the conversations that one preschool teacher orchestrated with her 10 students in order to co-construct sociomathematical norms, or norms for thinking and reasoning mathematically (Lopez & Allal, 2007). I observed and took field notes during all of the math lessons in this classroom that occurred over the course of 12 weeks. The teacher taught math lessons every Thursday and some Wednesdays. I interviewed the teacher at the end of each week to gain insight into her perspective on the lesson proceedings. I interviewed a few children during the first two weeks, but found that the children had difficulty answering my questions productively. Thus, after the second observation, I only interviewed the teacher.

From the beginning of the study, I was interested in the co-construction of sociomathematical norms in the classroom that took place through conversations. However, as I collected and analyzed my data, I came to focus even more narrowly on this aspect of the lessons. Going into the study, I posed three research questions:

1. What sociomathematical norms does one teacher and his/her students co-construct through conversations in the classroom, particularly related to numbers and operations?

2. How do linguistic differences shape two individual students’ experience of math lessons?
3. What specific strategies does the teacher use to extend conversations and include all of the students?

However, as the data collection and analysis moved forward, I made changes to these initial research questions to focus them more sharply on the co-construction of sociomathematical norms. As I collected data, it became clear that the concept of sociomathematical norms was a practical means of organizing and interpreting the data. This realization prompted me to revise all three questions as well as to add an overarching question.

My first research question pertained to what sociomathematical norms one teacher and her students co-constructed through conversations in the classroom, particularly related to numbers and operations. However, it soon became clear that more important than what sociomathematical norms were co-constructed was the role that these sociomathematical norms played in shaping subsequent conversation. As I answered this latter question, I came to answer the former question as well. With the “how” also came the “what”. Furthermore, with regard to this initial question, it became clear that the most productive conversations in Chen’s classroom did not always pertain to numbers and operations, but to premathematical topics that provided context for subsequent conversations about numbers and operations. For example, Chen and her students talked about time, about money, and about various math games. The children shared their personal experiences in connection with these concepts. Only after making these personal connections to the topics were the children ready to engage in productive mathematical conversations related to these topics.
The focus of the second question was on the experiences of the two students who were in the linguistic minority of the classroom. However, the collection and preanalysis of the data dictated that this question focus more on the primary concern of the study: the construction of sociomathematical norms. Thus, this question morphed into the question of how sociomathematical norms were constructed in a linguistically diverse classroom. This question still touched on the experiences of linguistically diverse students while remaining sharply focused on the construction of sociomathematical norms. This revision made for more streamlined data collection and analysis.

I also rewrote the final question of what conversation strategies the teacher used to extend conversation and include all students to focus more on the co-construction of sociomathematical norms. By asking what conversation strategies the teacher used to support the co-construction of sociomathematical norms, I also captured the ways she extended the conversation to include all students. Thus, by revising this question, I did not negate any of its initial meaning. The final guiding research question was as follows:

What is the role of sociomathematical norms in a preschool classroom?

This generated three subquestions:

a. How are sociomathematical norms constructed in a linguistically diverse classroom?

b. What are the obstacles to the co-construction of these norms?

c. What conversation strategies does the teacher use to support the co-construction of sociomathematical norms?
Overview of the Study

Much research has been devoted to the importance of engaging K-12 students in mathematical conversations. Another body of research has shown that preschool-age children are ready to learn more sophisticated math concepts than previously thought. However, little research has examined the kinds of conversations preschool teachers might engage their students in, nor the effect these conversations might have on preschoolers’ learning.

At the outset of the study, the participating teacher verbalized her belief that conversation was important in all academic areas, math included. Chen considered herself to be a constructivist teacher. She stated before the study that she possessed some skill in orchestrating conversation between children. However, when the study began, I found that Chen taught using an exclusively teacher-directed style (Tzuo, 2007). Chen herself noticed the discrepancy between her self-description and her actual teaching style once she had the chance to review the transcripts of her lessons, which I provided for her. Cook, Smagorinsky, Fry, Konopak, and Moore (2002) reported that some teachers believe they should be teaching in a constructivist style, but have trouble understanding what this practice should look like. Chen faced this dilemma. Part of this problem stemmed from the fact that constructivism is a theory of learning and not a theory of instruction.

Over the course of the study, Chen was successful in her own eyes at reconciling her practice with her philosophy. She gained skill and practice in engaging more of the children in extended conversation. In fact, Chen included all of the children in
conversation to some extent. As I described in Chapter 4, Chen succeeded in supporting more developmentally appropriate conversations.

The study yielded the following six, interconnected conclusions. In summary, I concluded that a teacher and her students can successfully overcome obstacles to conversation, successfully co-construct sociomathematical norms as a community, and engage in developmentally appropriate conversation. I found that this process can be mediated by the setting in which conversations took place. Finally, I found that students in the linguistic minority might require extra support to engage in these conversations. They might require extra support to join in on the co-construction of sociomathematical norms in the classroom micro-community (Yackel & Cobb, 1996).

Conclusions

1. Obstacles may initially prevent engagement in developmentally appropriate conversation and thus prevent co-construction of sociomathematical norms, but teachers and students can overcome these obstacles and converse productively.

2. Teachers can develop their own unique style of conversation with their students, overcoming obstacles to engage in developmentally appropriate conversation.

3. Preschool-age children can engage in extended, developmentally appropriate pre-math and mathematical conversations, leading to the co-construction of sociomathematical norms.
4. When a teacher focuses intentionally on giving students both guidance and freedom in conversation, these students may become more engaged in lessons and remember more of their content.

5. The setting in which the teacher and children interact may mediate the kinds of conversation that can take place and thus the kinds of sociomathematical norms that can be co-constructed.

6. Not all students will engage equally in conversations. Students in the linguistic minority may need extra support to engage in premath and mathematical conversations and thus to co-construct sociomathematical norms.

Overcoming Obstacles to Productive Conversation

When I entered into this study, I was prepared to document free-flowing mathematical conversations between the teacher and her students. National Council of Teachers of Mathematics (NCTM) (2000) recommended that mathematical communication take a central place in math lessons, for preschoolers as well as for students in K-12. I anticipated that in any preschool classroom, the teacher would experience successes and challenges in engaging her students in conversation. In fact, I found more obstacles than I anticipated stood in the way of productive conversation in Chen’s classroom (see Appendix E for a list of these obstacles). Because of these obstacles, the children were blocked from gaining verbal and mathematical skills that would enable them full participation in the classroom micro-culture (Lave & Wenger, 1999; Yackel & Cobb, 1996). These obstacles prevented the conversations from being
developmentally appropriate. My findings on this topic answered my second research subquestion: What are the obstacles to the co-construction of sociomathematical norms?

These obstacles were present even though Chen wanted to engage her students in conversation. I took note that even when a teacher intends to converse with his or her students, obstacles may prevent these conversations from taking place. These obstacles took many forms. Chen was at first hesitant to change her practice because she worried what the children’s parents might say if she taught in a different way. She was worried about what the other teachers and administrators would say. In their study, Lee and Ginsburg (2007b) found that both parents and teachers believed preschool-age children were not ready to learn math. Perhaps Chen’s worries about criticism from parents or fellow teachers were grounded in reality. She was afraid the children would not learn if she taught them in a new way. Chen worried that she would be unable to teach the material she had planned to teach if she allowed the children to converse freely. In fact, she did have to throw out most of her teaching agenda to follow children’s lead in conversation. Chen was worried that if she allowed children to speak off-topic, she would not be able to re-focus the children on a single math topic. Chen frequently spoke of the bravery she felt was required on her part to teach in a more reform-oriented style. I took note that teachers might have to draw upon their own resources to overcome the many obstacles they might face in attempting to engage their students in mathematical conversations. They could do so through reflection, planning, and experimentation (see Appendix F for a list of the ways Chen and the children overcame obstacles to conversation). Chen overcame these obstacles partially by compromising her
pedagogical agenda. Early in the study, Chen put her pedagogical agenda aside to give the children more freedom to converse. As time went on, Chen also developed nine distinct conversation strategies to both support children in speaking freely while continuing to focus them on the topic of her pedagogical agenda. See Appendix D for a list of these conversation strategies.

As Chapin and O’Connor (2007) reported, conversation strategies helped “establish conditions for respectful discourse” (p. 124). The children, for their part, showed through their behavior that they preferred to take an active role during conversations. Between Chen’s own resolve to overcome obstacles and these behavioral pressures from the children, Chen changed her conversational style dramatically from the beginning of the study to the end. As a result, she allowed children to become peripheral participants in her classroom microculture (Lave & Wenger, 1999; Yackel & Cobb, 1996). Conversation was now developmentally appropriate.

As the children conversed with the teacher and with each other, they co-constructed numerous sociomathematical norms. The development of conversation strategies was one method a teacher could use to overcome obstacles to conversation. This development was so successful that it seemed like a potential solution for any other teacher in a similar situation.

Developing a Unique Conversation Style

It cannot be taken for granted that a teacher possesses a unique style of engaging his or her students in conversation. Not all preschool teachers engage his or her students in mathematical conversation, even if they would very much like to. In this study, it was
clear at the outset that Chen did not engage her students in conversation. She lacked a style of conversation that was all her own. Initially, Chen experimented with changing her practice by giving the children complete freedom to speak about a topic in any way they wanted. As the weeks went by, Chen also practiced using conversation strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012) that allowed her to support students in contributing freely while still introducing material from her lesson plans. Tzao (2007) wrote that teacher-centered and child-centered instruction need not be mutually exclusive. Rather, teachers can strike a balance between the two. By phase three of the study, Chen had struck a balance between teacher-directed and child-directed instruction. My study suggested that through experimentation, teachers can develop a style of conversation that allows them to balance teacher-centered with child-centered instruction.

With this change in her conversation style, conversations between teacher and children and conversations between children and their peers were now developmentally appropriate. Chen and the children co-constructed sociomathematical norms that guided more equitable pre-math and mathematical conversations. In fact, I observed a feedback loop between sociomathematical norms and developmentally appropriate practice. The more developmentally appropriate conversation was, the more likely the group was to co-construct sociomathematical norms. Then, the more rigorously the group co-constructed sociomathematical norms, the more developmentally appropriate the conversation became.

My findings in this area enabled me to answer the overarching research question (What is the role of sociomathematical norms in a preschool classroom?) as well as the
third subquestion (What conversation strategies does the teacher use to support the co-construction of sociomathematical norms?). Broadly speaking, sociomathematical norms led to developmentally appropriate practice and vice versa. Conversation strategies aided and abetted the process of co-construction. As listed and described in Appendix D, I observed Chen using nine distinct conversation strategies.

In phase one of the study, Chen taught in a strictly teacher-centered style, delivering information and instructions, prompting children to finish her utterance, and prompting children to answer questions with short answers and without recounting personal stories. As the weeks went by, Chen changed her practice, allowing children to share personal stories, to speak without raising their hands, and to speak to each other. Ultimately, in the second phase of the study, Chen allowed the children to speak with much greater freedom.

In the third and final phase of the study, Chen was no longer simply imposing rules upon the children. Rather, she co-constructed sociomathematical norms (Lopez & Allal, 2007) with children. She included children in conversation to a radical extent, while still using conversation strategies to keep children focused on the topic Chen had chosen. She encouraged children to verbalize their reasoning, to make personal connections to material, to communicate with each other, and to talk through disagreements. Chen used conversation strategies (Chapin & O’Connor 2007; Temple & Doerr, 2012) to scaffold children in coming back to a central topic of discussion. Through these conversation strategies, Chen developed a unique style in supporting conversation. Through this unique style, Chen offered children access to the
knowledge and skills they needed to participate peripherally in the classroom micro-
culture (Lave & Wenger, 1999; Yackel & Cobb, 1996). While the reality was that
teacher and children co-constructed norms from the beginning of the study, this co-
construction became more deliberate and overt by the fifth week of the study.

Preschool-Age Children Are Capable of Engaging in Conversations

In their meta-synthesis of literature on mathematical conversations, Walshaw and
Anthony (2008) reported that K-12 children were capable of engaging in extended
mathematical conversations. They reported that these students were able to fine-tune
their mathematical thinking by verbalizing this reasoning. However, these authors did
not include studies on preschool-age students in their review. NCTM (2005) reported
that preschool-age children should learn to communicate mathematically. This report did
not offer any examples for what this communication should sound like at the preschool
level, but it suggested that preschool-age children can engage in productive mathematical
conversation. I found in my own study that preschool-age children, like older children,
were capable of fine-tuning their mathematical reasoning, especially through
conversation with the teacher or with their peers. Findings in cognitive science and
neuroscience have also shown that preschool-age children are ready to learn more
sophisticated math content than previously believed (Barth, La Mont, Lipton, & Spelke,
2005; Butterworth, 2005; Dehaene, 2011). These findings opened up the question of
what mathematical conversations should sound like at the preschool level. In my study, I
found that a preschool teacher was capable of engaging her ten students in
mathematically productive, developmentally appropriate conversation.
For example, in a lesson taught in the ninth week, Chen introduced a game in which cards with numerals had to be matched with cards showing the corresponding number of objects. Chen showed a card showing eight cars. She asked Mahesh how many objects he saw. Mahesh immediately answered that there were eight cars. Chen asked him how he got the answer so quickly. Mahesh said that he saw four and four, and added that he knew that equaled eight. Chen asked him how he was able to see the four and four so easily. Mahesh paused for a moment, then said, “You can kind of see that there are four on top and four on bottom. Then you can like add them together.”

Elodie added, “Yeah, I can see four on top and four on bottom! He’s right!”

The conversation continued in this vein as Chen showed the other cards. The children continued to subitize to arrive at the correct answer. As they went through card by card, Chen pressed the children to verbalize their reasoning for the answers they gave. Pressed by Chen to verbalize their thoughts, the children fine-tuned their reasoning just as Walshaw and Anthony (2008) reported K-12 children did. The children rose to the occasion each time that Chen initiated premath and mathematical conversation.

These preschool-age children not only engaged in conversation; they also initiated mathematical conversation themselves. In phases two and three of the study, the children engaged productively in numerous premath and mathematical conversations. My findings that the co-construction of sociomathematical norms enabled more children to engage in productive premathematical and mathematical conversation answered my overarching research question: What is the role of sociomathematical norms in a preschool classroom? These norms supported developmentally appropriate conversation.
Thus, I concluded that preschool-age children are capable of engaging in productive mathematical conversation under the right circumstances.

With Freedom and Guidance on How to Converse, Children Are More Engaged

While preschool-age children are capable of engaging in productive premath and mathematical conversation, they are not capable of doing so without the right freedom and guidance. In the first week of the study, Chen did not allow the children to contribute freely. They were permitted only to answer specific questions with a short answer. Further, they were not allowed to make personal connections to the topic because Chen was worried these off-topic contributions would derail the conversation. Craik (1934/2010) reported that the freedom to make personal connections enabled students to create new understandings. Most of the children’s verbal contributions were personal in nature. By forbidding this type of answer, she was stopping communication altogether.

Chen realized later that, as Craik had written, it was necessity for children to verbalize these personal connections in order to create new understandings. However, freedom was not enough for children to engage in productive conversation. According to NCTM (2000), children in lower grades need help making their ideas clear to their peers. Chen assisted them in doing so over the course of the study with the use of conversation strategies (Chapin & O’Connor, 2007; Yackel & Cobb, 1996). These conversation strategies modeled for children ways to think and speak mathematically.

During the first week, Chen did not allow the children to verbalize any personal connections to the material. The lesson for this week was on teen numbers. When Chen
assessed the children at the end of the lesson, most of the children answered her questions incorrectly. Only the two children Chen considered gifted answered her questions correctly. In the following lesson, Chen again assessed the children on the teen numbers. Most of the children once again gave erroneous answers. Reflecting upon the children’s difficulty in grasping her content when she restricted their participation, Chen resolved to let the children contribute more verbally to conversation, especially by making more personal connections.

In the next lesson, which was on time, Chen experimented by allowing just a few children make personal connections to the content. In this lesson, Chen asked the children how they knew what time it was. Mahesh told a story about his brother’s alarm clock. Later, when Chen asked the children to offer vocabulary words on time, Mahesh remembered the words “digital alarm clock”. Other children who had been allowed to tell a personal story remembered the time-related word associated with their story. Chen took note of this and in subsequent lessons allowed children even more freedom to tell their personal stories. As a result, the children remembered far more of the lesson content.

Starting in the third phase of the study, Chen began to using a variety of conversation strategies (Chapin & O’Connor, 2007; Temple & Doerr, 2012) to structure conversations. For example, she requested for children to clarify their contributions, to answer other children, to explain their reasoning, to add onto each other’s contributions, or to repeat another child’s utterance. She also summarized the conversation content intermittently to keep the children focused on the topic. She revoiced and recasted
student contributions. All of these conversation strategies served to focus children on the conversation at hand. Over the course of the third phase, the children appeared to internalize these conversation strategies and began to converse differently as a result. For example, they explained their reasoning without being prompted. They also added onto other children’s contributions. In these and other examples, the children’s conversation style changed as Chen exposed them to conversation strategies. These findings answered the first and third research sub-questions.

The children appeared far more engaged in the lessons especially in the third phase of the study when Chen gave them freedom to speak and when they also had acquired some conversation skills as a result of exposure to Chen’s conversation strategies. As Chen opened the floor to a variety of topics, more of the children engaged in conversation, including students in the linguistic minority. One important clue that children were more engaged was the decrease in misbehavior that occurred as the weeks passed. Over the course of phase one of the study, Chen disciplined the children 38 times. In phase two, she disciplined children only 27 times. In all of phase three, which was far longer than the other two phases, Chen only had to use discipline 26 times. This decrease in misbehavior in connection with the documented increase of engagement in conversation pointed to the fact that Chen’s students were more engaged once they were allowed more freedom to speak and once Chen had given them tools for productive conversation. I concluded that children need the right balance of freedom and guidance to engage in maximally productive conversation.
Students in the Linguistic Minority Need Extra Support to Engage in Conversations

Not all students start off on equal footing in classroom conversations. Students in the linguistic minority need extra support in order to access conversation equitably. The two students in this study who were in the linguistic minority were Aarav and Forrest. Aarav spoke mostly Hindi at home and Forrest spoke Mandarin. In phase one of the study, Aarav and Forrest were only called on to speak in the whole group setting one time each. The reason Chen did not call on them more often was that it was her habit to call only on students who were raising their hands. Further, during this phase, neither Aarav nor Forrest spoke without being called on. Barletta (2008) and Lagrander and Reid (2000) reported that children who needed the most practice speaking academic language often received the least. This was definitely the case at the beginning of my study of Chen and her students. By only calling on children who were raising their hands, Chen was not offering equitable opportunities for children to move into more central participation in the classroom micro-culture (Lave & Wenger, 1999).

As Chen reviewed the transcripts early in the study, she made note that she was not including these two students in conversation. Along with other changes she decided to make to her practice in phase two of the study, Chen decided to call on children even when they did not have their hands raised. Furthermore, in the second phase, Chen made her expectations for classroom conversation more explicit, thus giving children in the linguistic minority a better chance of being successful in joining in on these conversations (Moschkovich, 2002).
In phase two of the study, Chen called on the students in the linguistic minority more often. These children began to contribute more. In fact, as time went on, they spoke on several occasions without being called on. Chen believed that by contributing when she called on them, the children gained precious exercise in speaking in front of the group. She believed this practice gave them confidence to subsequently speak without being called on. In the third phase of the study, Aarav and Forrest engaged in some extended conversation with both the teacher and with peers. Thus, Chen partially disrupted the cycle of inequality (Barletta, 2008; Lagrander & Reid, 2000). It is important to note that even by the end of the study, these students still joined in conversation less than their linguistic majority peers. It is clear that students in the linguistic minority need more support in conversation than do their linguistic majority counterparts. These findings helped to answer the overarching research question as well as the second and third sub-questions.

The Setting Exerted Influence on Conversations

Different settings are established within the same classroom. Teachers often organize their students into a whole group, small groups, or in pairs. These settings make different kinds of conversation possible. The types of conversation and thus the kinds of sociomathematical norms constructed were mediated by the setting in which the conversations occurred. My findings on how setting exerted influence on conversation contributed to answering the overarching research question as well as the first (How are sociomathematical norms constructed in a linguistically diverse classroom?) and second research subquestions. Chen taught the children in three settings: the whole group setting,
the small group setting, and in pairs. It emerged from the data that Chen valued the whole group setting more than she did the other settings. Although she was aware that the least conversation took place in this setting, she felt that the conversations that did occur in this setting were highly valuable. She felt that this conversation was the most valuable because she was present in these conversations to scaffold it to make it more mathematical. Thus, these conversations were more focused on premath and mathematical material than were conversations that took place in the other settings.

Chen considered the conversation about money that took place in the fifth week of the study to be her most successful and exciting. In this conversation, the children cleared up many of each other’s misconceptions about money. They also solved a mathematical problem. The children talked as much as Chen did in this conversation. When she spoke, Chen did so to use conversation strategies such as revoicing, recasting, and asking children to evaluate one another’s contributions. Although in the other settings, the children did more of the talking, their conversation in those contexts were not as focused or as mathematical.

In the small group setting, children enjoyed more opportunity to speak because there were less children in the group to compete with. Chen felt less pressure to deliver information in this setting because it felt less official to her than did whole group lessons. Additionally, she felt it was a time when the children could practice conversing more intensively with each other, even if the content of their conversations were less on target in terms of her pedagogical agenda. Conversely, when Chen wanted to engage in more conversation with an individual, she was freer to do so in this setting than in the whole
group setting where the children were all waiting for their turn to speak. Thus, different opportunities for conversation were afforded by this setting. The data showed that children, especially those in the linguistic minority, enjoyed unique opportunities to verbalize their reasoning and to clear up one another’s misconceptions in this setting. Thus, the setting afforded different opportunities for co-construction of sociomathematical norms.

In the pair work setting, Chen intervened the least out of all the settings. For the most part, she only intervened on the conversations in this setting to clarify for her own understanding what the children were talking about. Thus, the children enjoyed the most opportunity for conversation in this setting. However, much of their conversation was imaginative or consisted of personal stories. Because Chen did not intervene to scaffold premathematical or mathematical talk, little of the conversation was strictly mathematical. It is notable that the students in the linguistic minority spoke most often in this setting. Chen believed that this setting helped to buttress their confidence so that they might contribute more in the other settings as well. Thus, even if their conversations were not related to math, the practice linguistic minority students enjoyed with their peers in this setting may have increased their confidence in speaking mathematically in other settings.

Although most conversations in this setting were imaginative or personal in nature, some important sociomathematical norms were co-constructed in this setting. For example, in the sixth week of the study, Mahesh and Elodie played a coin-matching game in the pair work setting. Elodie matched the wrong coins to the picture over the course of
this game. Mahesh was thus pressed to verbalize his reasoning to Elodie to help her match the coins correctly. He explained to her that the coins had to be the same size as the circle on the mat, and the number in the circle had to match the value of the coin. Through their conversation, Mahesh and Elodie co-constructed several sociomathematical norms, including speaking directly to a peer, talking through a disagreement, and clearing up a peer’s misconception.

I concluded that different classroom settings made different opportunities for conversation available. The whole group setting allows the teacher to employ various conversation strategies. Also in the whole group setting, children are free to converse with more of their peers. In the small group setting, the teacher is able to engage in more focused conversations with individual students. Also in the small group setting, students in the linguistic minority are more likely to converse. In the pair work setting, children are most likely to converse. In this setting, the children are capable of engaging in concentrated conversation with each other. However, they are not as likely to benefit from the teacher’s conversation strategies in this setting because it is more independent.

Linguistic Minority Students Needed Extra Support

In my final conclusion, I have noted that students in the linguistic minority do not engage as frequently in premath and mathematical conversations as do their linguistic majority counterparts. It is not clear why this is the case, but according to van Kleeck (2014), students in the linguistic minority are less likely to possess the language of specific academic subjects. In my study, it appeared that they did possess this language but did not feel as confident in using it as did their linguistic majority peers. As the study
unfolded and Chen changed her practices, students in the linguistic minority contributed more often and with deeper comprehension. In the first phase of the study, I only observed the two linguistic minority students to contribute one time each. Chen did not offer these students any extra support in participating during the first phase. In the second phase, however, Chen went out of her way to call on these students. She also relaxed rules so that children could answer in any way that they wanted rather than answering according to Chen’s strict rules. Under these circumstances, the students in the linguistic minority contributed more often. They answered questions and shared some personal connections during this phase.

These children began to contribute significantly more during the third phase of the study, when Chen and the children changed their practices even more dramatically. As Chen engaged the children in more productive conversation, the group began to co-construct sociomathematical norms. These norms allowed children to engage more freely and more deeply within liberal parameters set by Chen. In this phase, Chen used conversation strategies to scaffold the development of sociomathematical norms. For example, she requested for children to clarify their contributions, to answer other children, to explain their reasoning, to add onto another child’s contribution, or to repeat another child’s utterance. She also summarized the conversation content intermittently to keep the children focused on the topic. She revoiced and recasted student contributions. All of these conversation strategies served to support co-construction of sociomathematical norms.
Chen used these conversation strategies and other teaching strategies in the third phase of the study. With the assistance of these strategies, the group co-constructed norms for verbalizing their reasoning, for making personal connections, for speaking to each other, for asking each other questions, and for clearing up one another’s misconceptions. Although the students in the linguistic minority never contributed as often as the linguistic majority children, they did engage in co-constructioning all of the norms at different points of the third phase of the study. The data answered my first subquestion. Sociomathematical norms were co-constructed in this linguistically diverse classroom with specific supports from the teacher. It is important to note that, although Chen succeeded in orchestrating conversations that were developmentally appropriate for the other students, she never succeeded in orchestrating conversation that was perfectly equitable for all students.

During interviews, Chen pondered how she could involve these students in the linguistic minority in more conversation. She made note that they conversed almost as much as their peers did in small group and pair work time. Their participation in smaller settings signaled to her that Forrest and Aarav were capable of participating verbally in premath conversations. However, the kinds of communication expected of them in whole group settings were more technically mathematical. It seemed that their command of the English language was sufficient for chats with peers that took place in small group and pair work settings. However, they appeared to lack the tier two vocabulary words necessary to take part in more technically demanding premath and mathematical conversations (Biemiller, 2012).
Chen found that with extra support, she was able to bring these two students into more of the whole group conversations. She used conversation strategies and she also offered them possible answers to choose from when they appeared at a loss for words. Overall, Chen made much progress in engaging these students in premath and mathematical conversations. At the same time, she expressed the feeling that she needed to keep brainstorming ways to support future students in the linguistic minority so that they could participate as much as the students in the linguistic majority. My conclusion was that, even after all of Chen’s efforts to make conversations equitable for all, these efforts still fell short in this capacity. Students in the linguistic minority needed extra support to engage in premath and mathematical conversations. I concluded that even with support in conversing in developmentally appropriate ways, students in the linguistic minority may still be left out of the conversation. More research is needed for the educational community to understand how to support students in the linguistic minority in mathematical conversations.

Implications for Teaching

A number of important implications for instruction emerged from the data in this study. The first implication is that preschool-age children should be engaged by their teachers in premath and mathematical conversations. Developmentally appropriate premath and mathematical conversation leads to the co-construction of sociomathematical norms, which leads these young students to internalize math material. NCTM (2000) wrote that preschool-age children clearly benefited from engagement in mathematical conversations, but these authors did not describe what
practices teachers should employ to engage them in these conversations. In my study, I described a number of conversation strategies that preschool teachers can use with their students to promote complex mathematical conversations.

Klibanoff, Levine, Huttenlocher, Hedges, and Vasilyeva (2006) found that teachers’ mathematical inputs positively affected preschool-age students’ math scores. In my own study, I found that students benefited from the teacher’s use of conversation strategies. The teacher carefully tailored conversation strategies to the needs of her 10 preschool-age students. With the use of these conversation strategies, conversations became increasingly complex and productive. More students engaged in conversation. Other preschool teachers should make use of these conversation strategies. Additionally, they should design their own conversation strategies that meet the needs of their particular students as my participating teacher did.

Other preschool teachers can learn from my participating teacher’s practice of engaging students in co-construction of sociomathematical norms. Preschool teachers should encourage their students to talk through disagreements and to justify their reasoning. They learned to focus on one topic at a time, holding off-topic comments for later in the conversation. Preschool teachers should help students learn to clear up one another’s misconceptions about premath and mathematical concepts. All of these sociomathematical norms were co-constructed over the course of the twelve weeks by the teacher and students together. These discoveries could change the perceptions of early childhood teachers and caregivers that preschool-age children are not yet ready to learn math (Lee & Ginsburg, 2007a). Other teachers could engage their students in the co-
construction of these sociomathematical norms. Chen achieved this goal with a balanced conversation style, allowing the children both freedom and guidance. Other teachers can take inspiration from Chen’s accomplishment and strike a balance between a teacher-directed and a child-directed conversation style.

Preschool teachers reading this study will benefit from Chen’s struggles as well as her successes as she went through the process of incorporating more conversation into her math lessons. As Cook and colleagues (2002) reported, some teachers want to teach in a constructivist style but lack the training in how to go about teaching in this style. Chen faced many obstacles as she endeavored to converse with her children. She also faced many fears even as she moved forward in developing her unique conversation style. Preschool teachers can take inspiration from Chen’s story. In spite of obstacles and fears, Chen was able to develop a style of mathematical conversation that was all her own. Other teachers of preschool-age children can use the techniques Chen developed, or develop their own unique style just as she did.

Professional learning communities, coaches, and other instructional leaders can learn from the strategies I used with Chen. These strategies brought about rapid and significant change in her practice. These professionals can support teachers by videotaping their lessons and showing these recordings to the participating teacher, or they can provide a transcription of the teacher’s lesson in order for the teacher to study it. In either case, the teacher would gain the opportunity to reflect on his or her practice and perhaps make the kind of rapid change that Chen made. This hard evidence of her practice was powerful in causing Chen to reflect upon and change her practice. It should
be noted that Chen was highly motivated to change with or without my support. Thus, I would recommend that highly motivated teachers be chosen for this experience.

A number of forces were acting on Chen over the course of the study that prompted her to change her instruction. She remembered her training in her master’s degree program. She was inspired by speaker who visited during a recent in-service. Chen felt safe in incorporating more conversation into her practice because she knew the administration had approved my study, and that my study was officially on conversation. I shared feedback with Chen on one pivotal occasion and subsequently shared a study with her on teacher conversation strategies. Finally, Chen made careful study of the transcripts and made notes in the margins to help plan the changes she would make to her instruction. All of these forces were at work to influence Chen to change her practice quite rapidly.

Limitations of the Study

An important limitation to this study was my involvement in the events of the classroom. First of all, my presence palpably changed Chen’s decisions and behavior. She confided in me during interviews that she felt protected from the censure of the administration because she was participating in an official doctoral level study. Because of this protection, Chen was emboldened to make changes to her practice that she would not have dared to do alone.

Another important dimension of my involvement lay in my comment to her about conversation strategies I observed her using. My introduction of the research concept of conversation strategies led Chen down a dramatically different road than she may have
taken in my absence. Initially, I had hoped to participate with Chen as coresearchers. However, Chen opted to work independently to change her practice. In offering feedback to Chen in this single instance, I changed the results of the study. The outcomes would have been different if I had not influenced Chen’s thinking. However, I saw fit to point out to Chen what she was doing because I was not introducing any new ideas. I only indicated something that Chen was already doing. I pointed out to Chen that she was using two research-based conversation strategies. Chen then requested to see the research on conversation strategies. I provided her with one study. Chen used this research as a jumping-off point to develop her own array of conversation strategies all on her own. Chen benefited greatly from this intervention on my part, and she was thankful for my input. It was worthwhile for both myself and for Chen’s practice for me to have pointed out her use of conversation strategies. In this single instance, we acted as coresearchers in an otherwise purely ethnographic study.

Another important limitation of this study was that it was conducted in a private early childhood learning center. The findings do not reflect the realities of teaching in a public prekindergarten. Teachers in public prekindergarten classrooms, subject to state-mandated evaluations, must teach according to state-mandated curricula. It is not clear that a teacher in a public setting would have the freedom to change his or her practice as dramatically as Chen did. Public school teachers may not have as much freedom to experiment with their practice.

A third limitation was that I did not observe all of the mathematical conversations that took place in Chen’s classroom. I observed all of Chen’s math lessons that she
taught once or twice a week. However, these lessons were not the only times during the
day when the children engaged in premath and mathematical conversations. My inability
to observe all of the premath and mathematical conversations that took place over the
course of the day served to limit my study. According to Chen, many significant
mathematical exchanges occurred as the whole group talked about the calendar during
morning meeting. I did not observe these morning meetings due to my own scheduling
limitations.

In addition to morning meeting times, mathematical conversation occurred
incidentally over the course of the day, especially during free choice time when the
children interacted with math-related objects their teachers placed in activity
centers. Such math-related items included rulers, a clock, a timer, and a toy cash register.
I caught just the end of some of these conversations when I arrived for each observation
because the children were finishing their free choice time right before their math lessons
began. Initially, I regarded this data to be significant to the study. However, I later
determined that I did not observe enough of these conversations to include a robust
description of them in my data analysis. My depiction of the mathematical life in this
classroom would have been more detailed had I been present for all of these
conversations.

A fourth limitation was that I only interviewed the teacher at the end of each week
and not after each lesson. Because most weeks, Chen only taught one lesson, I did
interview her after most lessons. Interviews were an invaluable method of ascertaining
Chen’s perspective on what had taken place during lessons. Chen’s reflections on her
own practice often took me by surprise. Her perspective on her own work and especially on what she perceived as her progress profoundly influenced my data analysis. However, without a follow-up interview, I had to rely on my own interpretations of Chen’s instructional choices. It would have been truer to the ethnographic nature of my methodology to capture Chen’s perspective of each and every lesson.

The fact that I did not interview the children as originally planned was a fifth limitation. After the first few math lessons that I observed, I interviewed several children. I asked them questions to ascertain what they had learned and how they perceived different aspects of the lesson. The children generally answered my questions with one word answers at best and with a shoulder shrug at worst. Perhaps a more skilled or creative researcher could have devised the kinds of questions that would have elicited lengthier responses from preschool-age children. After a few tries, I abandoned my attempts to interview the children. The study would have been more thoroughly ethnographic if I had captured the perspectives of all the participants on the activities I had observed.

A final limitation was that, for the most part, I only observed the conversations that took place in situations in which Chen was present. I observed all of the whole group conversations. However, when Chen broke the children into small groups or pairs, I only observed the small group or the pair that Chen happened to be working with. Much data were lost because I did not capture the conversations taking place in the other small groups and/or pairs. This data could have been captured if I had been working alongside coresearchers who could have videotaped these other conversations.
Recommendations for Further Research

I captured just one teacher’s efforts to include her students in pre-math and mathematical conversation. More research similar to my own is needed before generalizations can be made about how preschool teachers successfully engage their students in mathematical conversations. To this end, more research could be conducted on the kinds of conversation strategies that preschool teachers find most appropriate for the age group they are working with. It would be interesting to investigate how well the conversation strategies used by my participating teacher work in another preschool classroom.

Only two students in the linguistic minority took part in this study. More research is needed on students in the linguistic minority and their experiences accessing math instruction, and specifically in accessing mathematical conversations. Future researchers could examine conversation strategies developed by the teacher specifically with linguistic minority students and assess if the difference in the number of contributions made by linguistic minority and majority students in the classroom disappears.

In this study, I described how sociomathematical norms were constructed. However, further research is necessary to further examine the mechanisms underlying the co-construction of sociomathematical norms. It seems likely that they were co-constructed in more than the four ways I described. Co-construction of sociomathematical norms occurred in subtle, barely perceptible ways. More research is needed to specifically examine this process and to identify ways co-construction occurs.
This study was qualitative, and thus, the conclusions are not generalizable. More qualitative research is needed to make a case for any of the individual findings. Furthermore, quantitative research is needed to examine what kinds of conversation strategies lead to improvements in student math scores. Quantitative research is needed to examine what kinds of sociomathematical norms lead to improvements in student math scores.

Finally, I would recommend the participatory approach I ended up using. It was helpful to both Chen’s practice and my study to share the transcripts with Chen. It was fortuitous that I chose a participant willing to study these transcripts carefully in order to plan to change her practice. I recommend choosing a participant who wants to study to change his or her practice to make it more developmentally appropriate so that we can better understand what developmentally appropriate practice looks like in action.

My experience leads me to recommend collaboration between researcher and participant in which the researcher gives constructive feedback. I recommend giving transcripts to the participating teacher after each observation. Where appropriate, researchers can share research with participants to help them reach their own goals. Participants can be chosen who are willing to study the transcripts of their lessons for the purpose of learning and changing practice to be more productive.

Final Thoughts

This study began as an attempt to fill the need for research on mathematical conversations at the preschool level. I found that one preschool teacher went through a journey from teacher-directed to increasingly child-directed conversations. This teacher
went from terse, one-sided verbal interactions with her students to rich premath and mathematical conversations with her students. I found that as the teacher progressively experimented with various conversation strategies to help scaffold student conversation, these conversations became progressively more developmentally appropriate. I found that this teacher’s 10 students benefited tremendously from the teacher’s efforts to design her own unique style of conversation.

I played a role in the changes that took place in Chen’s practice through my mere presence and by pointing out to her that she was using two research-based approaches. However, Chen took strong initiative to develop her own more child-centered conversational style. Chen faced obstacles and also constant anxiety about departing from traditional math instruction. Her strenuous efforts in reflecting on her instruction and in conducting pedagogical experiments are motivational for other teachers struggling to incorporate more reform-based approaches into their practice. It is possible for a teacher to transform his or her practice even late in the year. This study serves as an example of a journey that any preschool teacher can take.
REFERENCES


Barth, H., LaMont, K., Lipton, J., & Spelke, E.S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences, 102*, 14116-14121.


APPENDICES
APPENDIX A

MERCER UNIVERSITY IRB APPROVAL
Thursday, January 25, 2018

Ms. Betsy Gansberg
Mercer University
TIR College of Education - Atlanta
3001 Mercer University Dr
Atlanta, GA 30341

RE: Mathematical Conversations in a Preschool Classroom (H1801002)

Dear Ms. Gansberg:

On behalf of Mercer University’s Institutional Review Board for Human Subjects Research, your application submitted on 03-Jan-2018 for the above referenced protocol was reviewed in accordance with Federal Regulations 21 CFR 56.110(b) and 45 CFR 46.110(h) (for expedited review) and was approved under category(ies) 06, 07 per 63 FR 60164.

Your application was approved for one year of study on 25-Jan-2018. The protocol expires on 24-Jan-2019. If the study continues beyond one year, it must be re-evaluated by the IRB Committee.

Item(s) Approved:
New student application for ethnographic research study using observations of math lessons in a preschool classroom, interviews of the teacher and ten of students to help teachers, researchers, and policy makers discover whether or not preschool-aged children ready to reason mathematically, especially via conversations.

NOTE: Please report to the committee when the protocol is initiated. Report to the Committee immediately any changes in the protocol or consent form and all accidents, injuries, and serious or unexpected adverse events that occur to your subjects as a result of this study.

We at the IRB and the Office of Research Compliance are dedicated to providing the best service to our research community. As one of our investigators, we value your feedback and ask that you please take a moment to complete our Satisfaction Survey and help us to improve the quality of our service.

It has been a pleasure working with you and we wish you much success with your project! If you need any further assistance, please feel free to contact our office.

Respectfully,

[Signature]

Ava Chambless-Richardson, Ph.D., CPR, CIM
Associate Director of Human Research Protection Programs (HRPP)
Member
Institutional Review Board

"Mercer University has adopted and agrees to conduct its clinical research studies in accordance with the International Conference on Harmonization's (ICH) Guidelines for Good Clinical Practice."
APPENDIX B

TIMELINE OF OBSERVATIONS
Table B

*Timeline of Observations*

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Topic of Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thursday March 1</td>
<td>Teen numbers</td>
</tr>
<tr>
<td>2</td>
<td>Wednesday March 7</td>
<td>Time</td>
</tr>
<tr>
<td>3</td>
<td>Thursday March 14</td>
<td>Time</td>
</tr>
<tr>
<td>3</td>
<td>Thursday March 15</td>
<td>Time</td>
</tr>
<tr>
<td>4</td>
<td>Thursday March 22</td>
<td>Time</td>
</tr>
<tr>
<td>5</td>
<td>Thursday March 29</td>
<td>Money</td>
</tr>
<tr>
<td>6</td>
<td>Wednesday April 4</td>
<td>Money</td>
</tr>
<tr>
<td>6</td>
<td>Thursday April 5</td>
<td>Money</td>
</tr>
<tr>
<td>7</td>
<td>Thursday April 12</td>
<td>Marshmallows and spaghetti noodles</td>
</tr>
<tr>
<td>8</td>
<td>Thursday April 19</td>
<td>Light and shadow</td>
</tr>
<tr>
<td>9</td>
<td>Wednesday April 25</td>
<td>Making ten</td>
</tr>
<tr>
<td>9</td>
<td>Thursday April 26</td>
<td>Making ten</td>
</tr>
<tr>
<td>10</td>
<td>Thursday May 3</td>
<td>Manipulative centers including cuisenaire rods</td>
</tr>
<tr>
<td>11</td>
<td>Thursday May 10</td>
<td>Addition board game</td>
</tr>
<tr>
<td>12</td>
<td>Thursday May 17</td>
<td>Manipulative centers</td>
</tr>
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APPENDIX C

TIMELINE OF TEACHER INTERVIEWS
Table C

**Timeline of Teacher Interviews**

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Main Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Saturday</td>
<td>Obstacles to conversation, children’s off-task behavior, “striving for five”, getting through lesson material, not knowing where to start to promote conversation, the problem of stopping conversation</td>
</tr>
<tr>
<td></td>
<td>March 3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Saturday</td>
<td>Obstacles to conversation, children not retaining information, changing the rules, engagement of students in the linguistic minority, teacher bravery</td>
</tr>
<tr>
<td></td>
<td>March 10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sunday</td>
<td>Changes to the rules for conversation, anxiety about changing rules, teacher bravery, children learning new rules, teacher discipline of children</td>
</tr>
<tr>
<td></td>
<td>March 18</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Saturday</td>
<td>Changes to the rules, the revelation that Chen used certain conversation strategies, anxiety about changing rules, teacher bravery, children learning new rules, the importance of premath conversations</td>
</tr>
<tr>
<td></td>
<td>March 24</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Saturday</td>
<td>Discussing conversation strategies as described in the literature, using student contributions, improvement in student engagement, engagement of students in the linguistic minority, teacher anxiety, teacher surprise at student abilities in conversation</td>
</tr>
<tr>
<td></td>
<td>March 31</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Sunday</td>
<td>Developing her own conversation strategies, continued obstacles, overcoming obstacles, using student contributions, teacher surprise at student abilities, engagement of students in the linguistic minority, plans for small group work</td>
</tr>
<tr>
<td></td>
<td>April 8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Sunday</td>
<td>Conversation strategies, norms for conversation, using visual aids and manipulatives, overcoming obstacles, finding the hidden reason behind student comments, plans for small group work</td>
</tr>
<tr>
<td></td>
<td>April 15</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Saturday</td>
<td>Conversation strategies, norms for conversation, teacher bravery, finding the hidden reason behind student comments, the benefits of small group work</td>
</tr>
<tr>
<td></td>
<td>April 21</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Saturday</td>
<td>Conversation strategies, forgetting to use conversation strategies during small group work, norms for conversation, students’ progress in conversation, the benefits of small group work, facilitating instead of dictating</td>
</tr>
<tr>
<td></td>
<td>April 28</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Sunday</td>
<td>Conversation strategies, norms for conversation, using student contributions, balancing teacher-directed with student-directed instruction, using manipulatives, plans for the future</td>
</tr>
<tr>
<td></td>
<td>May 5</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Sunday</td>
<td>Conversation strategies, norms for conversation, using student contributions, plans for the future, teacher bravery, plans for pair work</td>
</tr>
<tr>
<td></td>
<td>May 13</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Saturday</td>
<td>Conversation strategies, some students’ voices still missing, plans for the future</td>
</tr>
<tr>
<td></td>
<td>May 19</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX D

TEACHER CONVERSATION STRATEGIES
1. **Clarification Request.** Question intended to elicit more information from a child when his or her answer or explanation is not complete.

Example:
Emily: The building will stand more better if we put the stick here.
Chen: What is your reasoning, Emily? Why do you think that stick will help your structure stand?
Emily: Because...because...because...it will make it stand up straight.

2. **Request for Child to Answer Another Child.** Teacher request for one child to answer the question of another child.

Example:
Chen: Let’s set this timer for thirty seconds. Then we’ll all jump. Let’s see how many times we can jump in thirty seconds. Who wants to set the timer?
Nancy: (Raises her hand).
Chen: Nancy, come over and set my phone timer for thirty seconds.
Nancy: (Looks at the timer but doesn’t set it.) I don’t know how to do it. How do you do it?
Chen: Nancy is asking us how to set the phone timer. Who can answer her question?
Hannah: (Raises her hand).
Chen: Hannah, please come up and help Nancy set the timer.
Hannah: (Approaches and sets the timer correctly). I think you do this. (Hannah presses buttons on Chen’s phone).
Chen: Thanks for your help, Hannah. I think I’ll have to set it for you. But good job asking your friend for help, Nancy.

3. **Request for Child to Add On.** Teacher request for a second child to add onto the contribution of a first child when that first child’s utterance was incomplete or unclear.

Example:
Chen: To win this game, you roll the two dice and try to make ten….
Olivia: (Rolls dice)
Chen: How many did you get?
Olivia: I didn’t win.
Chen: What do you mean you didn’t win? What did you roll? Mahesh, can you add on to what Olivia said?
Mahesh: She said she didn’t win...I think she means she didn’t make ten. She rolled two and three. So she only got five.

4. Evaluation Request. Teacher request that a child evaluate another child’s contribution.

Example:
Chen: When we minus one number from another one, what do we call that?
Aarav: Um, a little bit....
Chen: So when you minus something, you go from a lot to a little bit? Is that what you’re saying?
Aarav: (Nods head).
Chen: Elodie, is Aarav correct? Is that what it means to minus one number from another?
Elodie: Um, yeah. But it’s called subtraction.
Chen: You’re right. That is another word that Aarav could use.

5. Repeat a Peer’s Utterance. Teacher request for one child to repeat another child’s utterance.

Example:
Chen: It’s Forrest’s turn to roll. Here, Forrest.
Forrest: (Rolls dice).
Chen: What did you get, Forrest?
Beth: Eight
Logan: He got eight.
Forrest: I got three and four. I got seven.
Chen: Nancy, can you repeat what Forrest just said?
Nancy: He said he got three plus four.
Chen: What did he say that equals, Logan?
Logan: Um, he said seven but it’s eight.
Chen: Actually, I asked you to repeat what he said because he was right. He rolled three and four and that makes seven.

6. Explanation Request. Teacher request for a child to explain the reasoning behind their answer.

Example:
Chen: If something at the bake sale costs fifty cents, but the customer doesn’t have fifty cents….Maybe they only have dollars, so they give us a dollar….What change should we give them back?
Logan: A nickel!
Mahesh: Two quarters!
Chen: Mahesh said we should give them two quarters. Can you explain why Mahesh?
Mahesh: Because it’s fifty cents. One quarter is twenty-five cents. Two quarters are fifty cents.

7. Revoicing. The teacher repeats a child’s utterance to offer it to the group for explicit consideration.

Example:
Chen: We want our spaghetti structure to be what?
Beth: Strong.
Chen: Yes, we want it to be strong. Is there another good word to use?
Logan: We want it to stand up.
Chen: We want it to stand up.
Hannah: It should be nice and sturdy.
Chen: Ooh, I like that word. Did you two hear that Hannah said we want the structure to be sturdy? That’s a good building word.

8. Recasting. The teacher repeats a part or all of a student’s contribution, adding or altering it to provide more technically accurate mathematical information.

Example:
Chen: What do we have to do to solve this problem?
Logan: We have to count down.
Emily: We have to take some away.
Chen: Logan and Emily are both right. Remember another way to say it though is that we need to use subtraction. Everyone say, “subtraction”.

9. Offering Possible Answers. The teacher offers possible answers for the children to choose from when she knows she or a child has asked a question that would be difficult for the children to answer.

Example:
Chen: How should we break up into groups? We want each group to have the same number of kids if possible. Is that possible? How many groups should we have? Two? Three? Four?
Emily: I think we should have three groups.
Chen: Ah, three groups. I think that’s a good answer. How many kids should be in each group now?
Hannah: I think we could have three kids in a group.

10. Summarizing Conversation. The teacher intervenes to summarize what the children have said thus far in the conversation. The teacher uses this strategy when the children are offering many good ideas but are going off in disparate directions.

Example:
Chen: What kinds of things do we do at seven o’clock, when most of us wake up?
Elodie: We go to school.
Logan: I go to my grandma’s house on the weekend.
Hannah: I went on vacation last week. We had to wake up to go….We went up in a plane.
Chen: Let’s stop for a second and go over what we’ve said so far. I heard good answers. Most days, we get up to go to school, like Elodie said. But we do different things on the weekend, like Logan said. And every once in a while, we go on vacation starting early in the morning like Hannah said. So let’s start with what Elodie said.
APPENDIX E

OBSTACLES TO CONVERSATION
Obstacles to Conversation

1. Teacher Wanted to Get Through Pedagogical Agenda. The teacher was given a curriculum by the administrators of the school. She was not obligated to follow it to the letter. However, she felt obligated to cover as much of the material as she could. This prevented her from giving the children free air-time to express their questions and ideas.

Examples:

a. Chen: (Turning the pages of a book on time). We’ve talked about small times like seconds, minutes, and hours. We’ve talked about bigger times like weeks, months, and years. Do you know there are even longer times than that? When say “a long time ago” we might be talking about when people lived in castles (gestures to the picture in the book).

Children: (All raising hands and making impatient sounds).

Logan: Miss Chen! I have something to say about that!

Chen: We can talk about that later. First I have to finish this book.

b. Chen: (Turning the page of the same book described above). There are reasons we have night and day and different seasons. The earth revolves around the sun and also rotates on its axis. I know those are some big words. Here, take a look at this picture.

Nancy: I have something to say about the sun!

Chen: You’ll have to wait until later to share your idea. We’re trying to get to a fun activity that you’re all going to like.

2. Teacher Feared Off-Topic Contributions. The teacher was afraid that if she allowed children to offer contributions that were off-topic, these contributions would derail the conversation. She was afraid she would lose control of the conversation and of the children’s behavior. As a result, she stopped children from sharing off-topic.

a. Chen: (During an interview). I’m just scared that they’re going to go off in a million directions and I won’t be able to bring them back in. You know how this age group is. You might start talking about time. Then someone will say that they saw a clock at Disney World. Then all of a sudden you’re talking about Disney World.

b. Logan: I want this to be a castle (referring to his small group’s spaghetti and marshmallow structure). Let’s get some Lego men to live inside of it. (Goes to get a Lego man). This guy is the king of this castle. It’s candy castle like the Candy Land game.
Chen: Logan, we can talk about that some other time. Right now I want you to talk about the reasoning you are using to complete this activity.
Logan: I don’t want to. I want to use this Lego man (starts to cry).
Chen: I’m sorry, Logan. You can say whatever you want about your structure. Just not about the Lego man right now.

3. Teacher Feared Student Chatter. Just as when children offered off-topic contributions, when they began to chatter, Chen feared she would lose control over their conversation and their behavior. As a result, she put a stop to this chatter as soon as it started.

Example:

Chen: What are some tools we can use to tell time aside from the clock?
Mahesh: The phone!
Logan: A timer!
Hannah: A sand timer!
Chen: You mean the hourglass? Like the one that we have?
Children: (Begin to chatter about the hour glass)
Logan: Miss Chen, I know where the hourglass is. I’ll go get it!
Chen: No, Logan. We’ll look at the hourglass later. Right now we’re just talking. I need you all to focus. Come back to my question. Is there anything else we can use to tell the time?

4. Children were Impatient to Contribute in Their Own Way. Children were often inspired by various topics to contribute in seemingly off-topic ways. They were especially motivated to make personal connections.

Examples:

a. Chen: Let’s talk about the number fourteen. The number fourteen is made when we add magic ten to…
Olivia: My cousin Marla is fourteen!
Chen: We’ll talk about that later. For now, who can tell me, what do you have to add to magic ten to make fourteen?

b. Chen: A long time ago, princes and princesses lived in castles in places like Europe.
Nancy: Miss Chen, I got a princess hat for Christmas. It’s blue with sparkles.
Chen: We can talk about that later. Right now, we’re talking about time. There will be time to tell personal stories later.
5. **Children Acted Out.** The children sometimes acted out when Chen stopped them from contributing. On other occasions, they acted out because they just didn’t want to take part in the lesson. These interruptions posed obstacles to conversation.

Examples:

a. Chen: Who can tell me what season comes after fall?  
Emily: In the fall, me and my mommy make a pile of leaves and jump into them.  
Chen: Emily, can you answer my question? We’ll tell stories later.  
Emily: (Loudly bursts into song) La la la la la!  
Chen: Emily, your friends would like a chance to answer this question. We can’t hear when you sing.  
Emily: (Lies down on the carpet and rolls around).

b. Chen: (Pointing to the hands on the clock) Who knows what we call these?  
Logan: (Gets up from the circle, goes over to the hundreds chart, picks up the pointer and starts counting on the chart by loudly banging the pointer on it.)  
Chen: Logan, please come back to the circle so we can talk about the clock.  
Logan: I don’t want to talk about the clock! I want to count to a hundred.

6. **Linguistic Minority Children Contributed Infrequently.** Aarav and Forrest, the two students in the linguistic minority, made fewer verbal contributions than did the other students who were in the linguistic majority. They typically only contributed when Chen called on them. Even when Chen called on them, they were reluctant to answer.

Examples:

a. Chen: What comes next, Bria?  
Bria: Two!  
Chen: Then what, Forrest?  
Forrest: (Does not answer)  
Chen: You can do it, Forrest.  
Forrest: Three.

b. Chen: Who can tell me how to make thirteen?  
Emily: Ten and one!  
Chen: Remember to raise your hands. Aarav, what do you think? How do you make thirteen?  
Aarav: (Doesn’t answer).  
Chen: Aarav, just give it your best guess.  
Aarav: Ten and three?  
Chen: That’s right!
APPENDIX F

WAYS TEACHERS AND STUDENTS OVERCAME OBSTACLES
1. **Teacher Accepted More Answers.** Chen realized that by strictly limiting the types of verbal contributions children could give, she was preventing children from engaging in conversation. She decided to accept children’s answers to a radical degree.

Example:
Chen: Why do grown ups have jobs?
Elodie: Yesterday I went to Walgreens with my mom. She bought me toys.
Chen: Hold on. I asked why grown ups have jobs. Then Elodie said her mom bought her toys at Walgreens. Do you know that Elodie’s mom had to go to her job to make money? Then she had money to buy Elodie toys.
Elodie: Yeah, my mom has a job. She goes every day. If she didn’t have money, she would just have change.
Chen: Change is money too. You can’t get any type of money without a job.

2. **Teacher Developed Conversation Strategies.** As Chen progressed through the three stages described in chapter four, she developed conversation strategies to orchestrate conversation. Instead of stopping children from contributing, she scaffolded their contributions so that conversation made more sense. The conversation strategy Chen used most often was that of *revoicing*.

Example:
Chen: We’re going to write all of our time words on this big sheet of paper….I have an idea for a word. We talked about the kind of clock that doesn’t show all the numbers like our toy clock does. It only shows the numbers for what time it is now.
Logan: Alarm!
Emily: Hands!
Chen: No, this is a type of clock.
Mahesh: Digital!
Chen: Ah, Mahesh just said digital! That’s just what I was thinking about. Raise your hand if you knew the words ‘digital clock’. (A number of children raise their hands).

3. **Children Persisted in Speaking.** Sometimes Chen interrupted the children’s conversation to ask a new question. The children, however, persisted in speaking in the vein they felt was important. This persistence taught Chen to listen more carefully to the children and to follow their lead.

Example:
Chen: Let’s talk about the bake sale….We had cookies, cupcakes, and some candy. What did these things cost?
Beth: (Without raising her hand) I know, Miss Chen! Some of them cost fifty cents.
Some cost a dollar.
Chen: How much money did we make by the end of the bake sale?
Logan: A dollar equals a hundred pennies!
Chen: Friends, I asked how much money we made at the bake sale.
Mahesh: (Addressing Logan) A dollar also equals four quarters.
Logan: I think it’s a hundred pennies.
Mahesh: I know that. But we didn’t make a lot of pennies at the bake sale. We made lots of quarters.
Chen: Logan and Mahesh are both right. A dollar equals a hundred pennies or four quarters. And Mahesh is right that we made more quarters than pennies.

4. Children Persisted in Speaking for Extended Times. In the beginning of the study, Chen tried to stop the children when they went on speaking for too long. The children, however, persisted in speaking for extended times. This persistence led to Chen’s realization that she should follow the children’s lead and allow them to speak for extended times.

Example:
Chen: How do you know what time it is? What tools could you use to find out?...
Nancy: I don’t know how to read the clock. My Nana wakes me up in the morning. I don’t like it. I don’t know why...I don’t know why...I have to wake up so early. I just have to. I’m tired because of Nana wakes me up too early. I tried to tell you before but you wouldn’t listen.
Chen: So...
Nancy: I’m not done! My mommy says I can’t sleep at school. She wants me to read a book at nap.
Logan: My mommy says I can’t nap either. I have to do puzzles. If I sleep at nap, then I can’t sleep at night.
Nancy: My Nana brings me to school really early. I get here first before everyone else. But I get picked up late. I don’t know why I have to stay at school so long.
Chen: Nancy had a lot to say! She had a lot to say about the idea of ‘early’ and the idea of ‘late’. Those are opposites. Everyone say ‘opposites’.
Children: Opposites.
Chen: If she arrives early and leaves late, does that mean she’s here for a short time or a long time?
Children: A long time!

5. Children Acted Out. Paradoxically, when the children acted out, this caused Chen to reflect on her practice. She realized that by limiting conversation, she was causing children to act out. As a result, she revised her practice so that it would be more developmentally appropriate.

Example:
Chen: The reason we have days and nights is because the earth is rotating on its axis.
Hannah: I have something important to say.
Chen: You’ll have to wait just a second. I have to teach you about this.
Hannah: (Turns her back to the circle and folds her arms in frustration).
Chen: Okay, Hannah. Turn back around and I’ll listen to what you have to say.
Hannah: (Turns back around and talks happily). A long time ago, there was something...I forget what it was called...but me and my daddy looked at the sun and the sun was getting covered up. If you don’t use the special glasses to look at the sun, it can hurt your eyes.
Chen: That was called an eclipse. You’re right, Hannah, there was an eclipse. It happened to the sun and lots of people used special glasses and other devices to look at it. That’s connected to what we’re talking about. That happens when the moon gets between the earth and the sun.

6. Children Internalized Sociomathematical Norms. By the third phase of the study, the students had co-constructed sociomathematical norms with the teacher. Their cooperation in this construction allowed them to internalize these norms. They engaged in conversation and rarely acted out. They spoke directly to each other during conversations. They talked through disagreements. They asked each other questions. And they verbalized their reasoning.

Examples:

a. Speaking directly to each other.
Chen: How will we begin counting how much money we made at the bake sale? We have all the money in this basket. How should we start?
Olivia: (Picks up a few pennies and starts counting). One, two, three, four…
Mahesh: Olivia, first we should make a pile of all the pennies and we can count those. We can make a pile of the dollars and count those in one pile.

b. Talking through disagreements.
Logan: (Putting coins on his game sheet willy-nilly).
Nancy: No, Logan, those coins don’t go there.
Logan: But I want to put them there.
Nancy: (Picking up one coin). But look. This coin is too big for this circle. I think this circle is for this coin. Look! (Putting a nickel on the circle).
Logan: Okay, okay.

c. Asking each other questions.
Beth: Our building is falling. What should we do?
Hannah: Hold it up while I stick more sticks in it.
Beth: (Holds the structure up while Hannah inserts more spaghetti noodles into the marshmallows).

d. Verbalizing reasoning.
Chen: What do you think will happen when we shine the light through these colored tiles?
Emily: I think it will make colors.
Chen: Why?
Emily: Because they’re different colors.
Chen: Is that the only reason?
Emily: Also they’re made of different stuff, so their shadows will look different.
APPENDIX G

PARENTAL CONSENT FORM
Mathematical Conversations in a Preschool Classroom

Parent or Guardian Informed Consent Form

Your child is being asked to participate in a research study entitled, “Mathematical Conversations in a Preschool Classroom”. The study is being conducted by myself, Betsy Gansborg. I can be contacted at 404-423-4908 or at Betsy.Gansborg@live.mercer.edu. The advisor overseeing this study is Dr. William O. Lacefield III who can be reached at 678-547-6335 or LACEFIELD_WO@mercer.edu. The results will be used to further my understanding of how one preschool teacher engages students in conversations about mathematics, and the mathematics learning the students experience as a result. Your child’s participation is voluntary. A decision to participate in the research will not affect his/her relationship with Dunwoody Prep, his/her relationship with other teachers, or his/her academic standing.

I. The purpose of my study is to explore:

This research study is designed to examine conversations that occur between the teacher and students during math lessons and the children’s subsequent math reasoning and knowledge of math concepts.

The data from this research will be used to extend research on academic conversations and how these contribute to student learning at the preschool level. Completion of this study is required for the fulfillment of my PhD program in Curriculum and Instruction at Mercer University.

II. Procedures:

If you allow your child to volunteer for this study, he or she will take part in math lessons just as he/she would normally would. I will observe these math lessons and periodically conduct brief interviews with the child to ascertain his or her understanding of the topics covered in the lessons. Your child’s participation will take approximately forty minutes to an hour and a half per week. This participation will consist of their engagement in their regular math lessons, plus another fifteen minutes or so every few weeks for a brief interview with me.

Your child will be asked to assent to participate in this research (Assent means that your child will be asked to voluntarily participate in this research). Your child will tell the teacher they want to participate by answering YES or NO after the teacher verbally reads to your child what the research is about and what he or she will be asked to do. The child will be observed during small group math lessons once or twice a week and will take part in a one-on-one interview with me, the experimenter, every few weeks.

Parent/Guardians who allow students to participate must:

Read and complete this consent form, giving permission for me (Betsy Gansborg) to observe your child in math lessons and to intermittently participate in a brief interview with me about the math content taught during the observed lessons.

MercerIRB
Approval Date 01/25/2018
Protocol
Expiration Date 01/24/2019

Rev. January 2017
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III. Potential Benefits to Students and/or Society

Some potential benefits for students and society include increased understanding of how one teacher engages her preschool-aged children in productive math conversations. Conversations have the potential to help children develop reasoning instead of learning correct answers by rote memory. Increased understanding of how such conversations work could help other teachers engage their students in more conversation as well.

IV. Potential Risk and Discomforts

There are no foreseeable risks associated with the study. Interviews with children will be short and infrequent. However, there is a chance the child will dislike to be interviewed, in which case the interviews with him or her will be terminated. If for any reason whatsoever, you as the parent would like for your child to be excluded from the study, you can let me know and I will remove him or her from the study immediately.

V. Withdrawal of Participation

Your child’s participation is voluntary. Your child will not be penalized or lose any benefits that he/she are otherwise entitled to if you decide that your child will not participate in this research project.

If your child decides to participate in this project, he/she may discontinue participation at any time without penalty or loss of benefits. You have the right to inspect any instrument or materials related to the proposal. Your request will be honored within a reasonable period after the request is received.

VI. Payment for Participation

Students will not be paid for their participation. There is no financial obligation for participants.

VII. Confidentiality and Data Storage

The confidentiality and privacy of the participants will be carefully preserved. The children’s real names will never be used in the data or in the final write-up. Furthermore, in the final write-up of the study, I will leave out descriptors that would enable readers to identify the school in which the study took place.

Data derived from observations and interviews will be stored only in my home and at the Atlanta campus at Mercer University. Data will be stored at Mercer University for three years after the study.

Observations will be videotaped and interviews will be tape recorded. However, no one will watch or listen to this media except for me, the experimenter. I will transcribe this media into print myself. This print material may be shared with the three professors on my dissertation committee. These professors are Dr. William O. Lacefield III, Dr. Jane West, and Dr. deb Rosenzein.

All information pertaining to the study will be kept confidential. Your child’s name will not be associated with his or her individual responses and will be identified only by an assigned coded number. At no time will your child’s name be associated with the results of the research or shared with parents or others. Any identifying information provided by your child will never be used as part of the research or associated with the results of the study.

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Your child’s responses will be stored in a locked location and will only be used for research purposes by Mercer University School. A number will identify the information that I collect from the observations of and interviews with your child. The list connecting participant numbers and names will also be kept in separate locked cabinets.

Questions about the Research

If you have any questions about the research, please speak with Dr. William O. Lacefield III. If you have questions at any time, you may contact him at 678-547-6335 or LACEFIELD_WO@mercer.edu.

You have been given the opportunity to ask questions and these have been answered to your satisfaction. If you agree to allow your child to participate in this research, please complete the information below:

I, __________________________, grant my child, __________________________, permission to participate in this research study.

Name of Parent or Legal Guardian Name of Child Participating in Study

Parent/Guardian Name (Print) Name of Person Obtaining Consent (Print)

Parent/Guardian Signature Person Obtaining Consent Signature

Date Date

Please return to Betsy Gansborg, the experimenter, as soon as possible.

In order to conduct this research, this project has been reviewed and approved by Mercer University’s Institutional Review Board (IRB). If you believe there is any infringement upon your child’s rights as a research subject, please contact the IRB Chair at (478) 301-4101. The IRBs are the governing bodies that are set in place to ensure responsible and safe conduct of research investigations.
Mathematical Conversations in a Preschool Classroom

Informed Consent

You are being asked to participate in a research study. Before you give your consent to volunteer, it is important that you read the following information and ask as many questions as necessary to be sure you understand what you will be asked to do.

Investigators
Betsy Lauren Gasborg, PhD candidate Tift College at Mercer University, PhD in Curriculum and Instruction
3001 Mercer University Drive, Atlanta, GA 30341, (678) 547-6330

Dr. William O. Lacefield, PhD (Advisor) Tift College at Mercer University, PhD in Curriculum and Instruction
3001 Mercer University Drive, Atlanta, GA 30341, (678) 547-6335

Purpose of the Research
This research study is designed to investigate ways one preschool teacher engages her ten students in conversations about mathematical concepts as well as the resulting mathematical understandings of the students.

The data from this research will be used to extend research on mathematical instruction at the preschool level, especially with regard to appropriate verbal communication during math lessons.

Completion of this study is required for the fulfillment of my PhD program in Curriculum and Instruction at Mercer University.

Procedures
If you volunteer to participate in this study, you will be asked to conduct math lessons as you would normally do under my observation. In addition, you will be asked to participate in one interview a week. This interview should take no longer than thirty minutes weekly.

Your participation will take approximately an hour to two hours per week for approximately four months.

This participation will include observations of the math lessons you teach to your ten students as well as the one weekly interview you will take part in with me.

Potential Risks or Discomforts
There are no foreseeable risks, discomforts, inconveniences, or costs associated with this research that you may encounter.

However, if you find that the study does inconvenience or harm you in any way, you can discontinue your participation at any time by informing me of your decision.

Potential Benefits of the Research
A possible benefit to you for participating in this study is the opportunity to reflect on your practice. Our conversations during interviews might allow you to reflect on your math instruction from a new perspective.

The potential benefits to the education research community as well as to society is increased understanding of how a teacher may engage his or her students in productive mathematical conversation.

Confidentiality and Data Storage
I will take several precautions to protect your privacy as I conduct this proposed study. I will not use your name in any of the data or in the final write-up. I will store the data only in my home and at Tift College in Mercer University. Confidentiality will be strictly maintained. Only I will watch or listen to video and tape recordings.

I will use and store the data in my home as well as in the office of Dr. William O. Lacefield III at Mercer University. The data must be stored at Mercer University for three years. The video and tape recordings will be used by me over the course of the approximately four months of the study as well as during the approximately three months after the study, during which time I will continue to analyze the data and complete a final report. After these seven months, I will destroy all of these video and tape recordings.

Participation and Withdrawal
Your participation in this research study is voluntary. As a participant, you may refuse to participate at any time. To withdraw from the study please contact myself at 404-423-4908 or Dr. William O. Lacefield at (678) 547-6355.

You can withdraw simply by asking me or my advisor. Note that because you will be described anonymously, you cannot withdraw after the data has been collected.

Questions about the Research
If you have any questions about the research, please speak with me or my advisor. My phone number is 404-423-4908 and my Mercer email is Betsy.Gansborg@live.mercer.edu. The advisor overseeing this study is Dr. William O. Lacefield III who can be reached at 678-347-6355 or LACEFIELD_WO@mercer.edu.

Audio or Video Taping
Please be advised that in signing this consent form, you consent to being videotaped as well as tape recorded. Your name will be used in the video and audio tapes. However, these recordings will be destroyed at the end of the data analysis period and before the publication of the final report. Further, no names will be used in the transcripts of these recordings, nor will names be used in the final write-up of the study.

This project has been reviewed and approved by Mercer University’s IRB. If you believe there is any infringement upon your rights as a research subject, you may contact the IRB Chair, at (478) 301-4101.

You have been given the opportunity to ask questions and these have been answered to your satisfaction. Your signature below indicates your voluntary agreement to participate in this research study.

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Rev. January 2017

Mercer IRB

Approval Date

01/25/2018

Protocol

Expiration Date

01/24/2019

Date

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